MATH 114 Spring 2022

George Mason University Korea

MATH 114 (Analytic Geometry and Calculus II) - Spring 2022 (On-Line/In-Person)

Syllabus

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Office</th>
<th>Tel</th>
<th>e-mail</th>
<th>Common Office Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr. C. Kim</td>
<td>Mason Bldg 555</td>
<td>5140</td>
<td><a href="mailto:ckim50@gmu.edu">ckim50@gmu.edu</a></td>
<td>Monday, Wednesday: 4:00-4:50; Thursday: 4:30-5:20 or by Appointment</td>
</tr>
</tbody>
</table>

Course Objectives: (by the Catalog http://catalog.gmu.edu/course-search/?subject=MATH)

1. To develop a facility with the concepts and techniques of differential and integral calculus.
2. The material covered will be: various methods of integration, conic sections, parametric equations, infinite series, and power series.
3. To provide a strong foundation in calculus as preparation for subsequent courses in mathematics and many other disciplines.
4. To improve the student’s analytic thinking, quantitative reasoning and problem-solving ability.

Learning Objectives: At the end of this course, the successful student will be able to

1. Develop analytical and numerical expressions using real problems.
2. Students are able to interpret quantitative information and draw inferences from them.
3. Given a quantitative problem, students are able to formulate the problem quantitatively and use appropriate arithmetical, algebraic, and combinatorial methods to solve the problem.
4. Apply appropriate mathematics principles to evaluate expressions, and find and test potential solutions.
5. Students are able to evaluate logical arguments using various calculus computing.

Textbook:


Other Resources:

1. The Math Tutoring Centre is located 6th floor, GMU building. Help is available on a walk-in bases with an appointment.

Teaching Mode: 4 + 0 hours of lectures per week (Classroom: G305)

Evaluation: (All of evaluations should be in-person except Test #1.)

- 18% Test #1 (100 minutes) 14:00 - 15:40, Mar. 21st, 2022. (on-line test, new material only)
- 18% Test #2 (100 minutes) 14:00 - 15:40, Apr. 18th, 2022. (in-class test, new materials only)
- 18% Test #3 (100 minutes) 14:00 - 15:40, May 16th, 2022. (in-class test, new material only)
- 27% Exam (150 minutes) 12:30 - 15:00, June 13th, 2022. (in-class test, cumulative)
- 9% Participation during classes: M/C Pop Quizzes, T/F Questions, Participation during the classes
- 10% Recitation with Tutor in Mathematic Tutoring Center or Instructor

---

1ver.2: Revised on February 19th, 2022 because of On-Line Instruction
2This openbook will be used for recommended practice problems.
3Download Link: https://openstax.org/details/books/calculus-volume-2
4There is a tentative schedule for tests below. You are responsible for keeping up with all information announced in the classroom and on Blackboard. There will be no makeup tests. You may replace your missed test grade with your final exam percentage.
Missed Tests or Final Exam

1. Students who are unable to be present for a midterm or for the exam due to illness, must contact the instructor by email or in person prior to the time of the evaluation or within the time period stated in academic policy.

2. Students who miss a test will NOT be given the opportunity to write a make-up test. If the final exam is missed, an INC may be given in accordance with the policies set out in the Calendar. Failure to provide the appropriate documentation in time will lead to a grade of zero for the missed evaluation.

Honor Code

1. It is mandatory that each student’s conduct is within the guidelines of the George Mason Honor Code.

2. Sharing information of any kind about tests, exams, quizzes will result at a minimum in a grade of zero for all parties involved.

3. Violations will also be reported to the university Honor committee where further consequences such as expulsion from the university may be incurred.


Evaluation Guideline

1. All of the tests will be closed-book and written without calculators or any other aids.

2. There will be no supplemental test.

3. Grades will be assigned as indicated in the George Mason Academic Calendar.

4. Any tests written in pencil or erasable pen are ineligible for remarking.

5. There is no intrinsic reason for giving a non-zero mark for an incorrect solution. Part marks (if any) are awarded entirely at the examiner’s discretion. If a test is submitted for re-marking, the whole test may be re-marked. The result may possibly be that the student receives a lower mark on any or all questions.

6. Students are responsible for completing all recommended homework in a timely fashion as the course progresses. Homework must be kept organized and legible as it may be recalled for marking as part of the student’s evaluation.

7. Talking to another student, glancing over another student’s paper or being caught with non-allowed materials during an evaluation may result in a zero mark for that evaluation and a record of academic misconduct lodged with the Registrar’s office.

8. During an evaluation sharing of pencils, pens or erasers is NOT PERMITTED and PDAs, phones and pagers must be turned off and out of reach.

9. During any evaluation, coats, jackets and bags must be placed out of reach.

Other Resources

The Math Tutoring Centre: The Academic Resource Center, GMUK is in the business of looking at your papers and problems to improve your academic achievement in the area of Writing, Mathematics, Accounting, Statistics, and Economics. You are invited to utilize the faculty and student tutor services at a variety of stages in your academic activities. They do help you become conscious of particular error patterns that emerge in your work. For more information, please contact Professor Eunmee Lee, director of Academic Resource Center (elee45@gmu.edu, office #621).
Disability Services

Reasonable accommodations are available for students who have a documented disability. Please contact Disability Services if you require accommodations: Office of Disability Services, Student Union Building I (SUB I), room 4205, Phone: 703-993-2474

Counseling & Psychological Service (CAPS)

1. (Fairfax Campus) Counseling and Psychological Services provides a wide range of free services to students. Individual and group therapy, workshops, online self-help, and community education programs are designed to enhance students' personal experience and academic performance. (703) 993-2380; http://caps.gmu.edu

2. (Songdo Campus) Individual counseling is available. You may schedule an appointment here: Appointments. Group counseling is available. If you are interested in group counseling, please email wellness@gmu.edu or call +82-32-626-6142 to schedule a consultation. Please be aware that you may be asked to reschedule your appointment due to time constraints if you are more than 10 minutes late. To schedule an online appointment: https://appointmentwithjan.as.me

Regarding electronic devices

Please be respectful of your peers and your instructor and do not engage in activities with electronic devices such as laptops, tablets, cell phones, etc. that are unrelated to class. Such disruptions show a lack of professionalism and it will affect your participation grade. Cell phones shall be set to silent/vibrate and placed out of sight when not used for class activities, and especially during exams and quizzes.

Diversity

You are expected to behave in accordance with the GMU Diversity Statement: http://ctfe.gmu.edu/professional-development/mason-diversity-statement/

Privacy

Students must use their masonlive email account to receive important University information, including messages related to this class. See http://masonlive.gmu.edu for more information.

Student Care

Mason Korea Student Care is committed to providing an empowering environment for all Mason students to seek support services and to explore healthy life choices. We offer educational programming, one-on-one consultations, and resources in the areas of disability services, sexual and interpersonal violence, financial well-being, substance use, and collegiate recovery. We also assist students in encountering barriers to personal success. Please email Mason Korea-related inquiries to spark214@gmu.edu.

For emergencies after the normal business hour (Mon-Fri 9 AM to 5 PM), you can contact 032-626-5119. NOTE: If you are reporting a situation involving an imminent threat of harm to self or others, please the Police at 119 first.
## Course Contents & Schedule

<table>
<thead>
<tr>
<th>Week</th>
<th>Dates</th>
<th>Topics</th>
<th>Text Book</th>
<th>Recommended Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0221</td>
<td>volume by slicing, shell</td>
<td>6.1-3</td>
<td>2.2 Ex (pp150) #63,75,79,85,87,93,105</td>
</tr>
<tr>
<td></td>
<td>0223</td>
<td></td>
<td></td>
<td>2.3 Ex (pp166) #123,127,133,135,143,146,159</td>
</tr>
<tr>
<td>02</td>
<td>0228</td>
<td>length, area exp, ln ft</td>
<td>6.3-6</td>
<td>2.4 Ex (pp180) #173,177,185,195,203</td>
</tr>
<tr>
<td></td>
<td>0302</td>
<td></td>
<td>7.1,7.3</td>
<td>2.7 Ex (pp230) #313,315,321,331,337,345</td>
</tr>
<tr>
<td>03</td>
<td>0307</td>
<td>by part</td>
<td></td>
<td>3.1 Ex (pp270) #9,13,17,23,29,33,37,49,51,57,61,65,67</td>
</tr>
<tr>
<td>04</td>
<td>0314</td>
<td>trig. Integral &amp; substitution</td>
<td>8.3</td>
<td>3.2 Ex (pp283) #75,77,83,85,89,93,95,99,101,105,107,111,113, 121,125</td>
</tr>
<tr>
<td></td>
<td>0316</td>
<td></td>
<td>8.4</td>
<td>3.3 Ex (pp296) #137,139,141,145,149,153,157,159,161,163,165,169,175,179</td>
</tr>
<tr>
<td>05</td>
<td>0321</td>
<td>Test #1</td>
<td>8.5</td>
<td>3.4 Ex (pp308) #197,199,201,203,205,209,211,213,217,225,227,229,235,237,243</td>
</tr>
<tr>
<td></td>
<td>0323</td>
<td>PFD</td>
<td></td>
<td>3.4 Ex (pp308) #197,199,201,203,205,209,211,213,217,225,227,229,235,237,243</td>
</tr>
<tr>
<td>06</td>
<td>0328</td>
<td>improper int separable DE</td>
<td>8.8</td>
<td>3.7 Ex (pp343) #349,351,357,359,363,365,369,373,377,381,387,389,395,399</td>
</tr>
<tr>
<td></td>
<td>0330</td>
<td></td>
<td>7.2</td>
<td>4.3 Ex (pp391) #125,129,131,135,137,139,141,149,151,155,159</td>
</tr>
<tr>
<td>07</td>
<td>0404</td>
<td>linear DE sequence 1</td>
<td>9.1-3</td>
<td>4.5 Ex(pp420) #215,217,219,221,225,229,231,243,245,247,249,251,261</td>
</tr>
<tr>
<td></td>
<td>0406</td>
<td></td>
<td>10.1</td>
<td>5.1 Ex (pp447) #5,9,11,15,17,25,29,33,35,37,39,43,47,49,51,53</td>
</tr>
<tr>
<td>08</td>
<td>0411</td>
<td>sequence 2 series</td>
<td>10.2</td>
<td>5.2 Ex (pp466) #73,75,77,81,85,87,89,91,95,99,101,103,105,107,109,113</td>
</tr>
<tr>
<td></td>
<td>0413</td>
<td></td>
<td></td>
<td>5.2 Ex (pp466) #73,75,77,81,85,87,89,91,95,99,101,103,105,107,109,113</td>
</tr>
<tr>
<td>09</td>
<td>0418</td>
<td>Test #2</td>
<td>10.3</td>
<td>5.3 Ex (pp482) #143,149,151,155,157,161,163,167,171,173,175,179,181</td>
</tr>
<tr>
<td></td>
<td>0420</td>
<td>Int test</td>
<td></td>
<td>5.3 Ex (pp482) #143,149,151,155,157,161,163,167,171,173,175,179,181</td>
</tr>
<tr>
<td>10</td>
<td>0425</td>
<td>dct</td>
<td>10.4</td>
<td>5.4 Ex (pp493) #199,201,205,207,211,213,217,219</td>
</tr>
<tr>
<td></td>
<td>0427</td>
<td>lct</td>
<td></td>
<td>5.4 Ex (pp493) #199,201,205,207,211,213,217,219</td>
</tr>
<tr>
<td>11</td>
<td>0502</td>
<td>ast</td>
<td>10.5-6</td>
<td>5.5 Ex (pp505) #253,255,257,259,263,265,267,269,271,275, 277,287,289,283, 299,301,303</td>
</tr>
<tr>
<td></td>
<td>0504</td>
<td>rot,rat</td>
<td></td>
<td>5.6 Ex (pp522) #323,325,327,329,333,337,341 343,345,349,351,355,359,363, 367,369,371,375</td>
</tr>
<tr>
<td>12</td>
<td>0509</td>
<td>power series properties</td>
<td>10.7</td>
<td>6.1 Ex (pp541) #9,11,17,19,21,25,27,31,37,39,41,45,47,51,53</td>
</tr>
<tr>
<td></td>
<td>0511</td>
<td></td>
<td>10.8</td>
<td>6.2 Ex (pp558) #67,71,81,83,85,87,89,93,95,97,99,101,105,107,109</td>
</tr>
<tr>
<td>13</td>
<td>0516</td>
<td>Test #3</td>
<td>10.9</td>
<td>6.3 Ex (pp578) #119,121,123,127,129,133,135,141, 143,147,149,153,157,159,171,173</td>
</tr>
<tr>
<td></td>
<td>0518</td>
<td>Taylor series</td>
<td></td>
<td>6.3 Ex (pp578) #119,121,123,127,129,133,135,141, 143,147,149,153,157,159,171,173</td>
</tr>
<tr>
<td>14</td>
<td>0523</td>
<td>applications</td>
<td>10.10</td>
<td>6.4 Ex (pp596) #177,179,183,187,191,197,199,201,205 207,209,211,213,215,221,223,227,229,231,247</td>
</tr>
<tr>
<td></td>
<td>0525</td>
<td>parametric curves</td>
<td>11.2</td>
<td>7.2 Ex (pp640) #69,73,77,83,85,89,91,101,109,115</td>
</tr>
<tr>
<td>15</td>
<td>0530</td>
<td>polar coordinates</td>
<td>11.3-5</td>
<td>7.3 Ex (pp660) #137,143,151,153,159,170,173</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.4 Ex (pp669) #193,199,205,209,213</td>
</tr>
</tbody>
</table>

**Final Exam**

- Dates: Four numbers mean months and days. The first two are months and two are days.
- There will be no class on 0309 (March 9th) and 0601 (June 1st) because of nationwide elections.
- GMUK will announce the final exam schedule later.

---

5All chapter, section and page references pertain to *Calculus, Volume 2, by Strang, Herman, et al* (2016)
Weekly Assignment

Weekly Assignment #01

1. FARBGE \( x = \cos y, x = \frac{1}{2}, y = \frac{\pi}{3}, y = \frac{7\pi}{3} \).

2. Find the volume in two different methods of the solid generated by revolving the plane region bounded by the equuations:

\[ y = \sqrt{x}, y = 2, x = 0 \]

revolved about

(a) the \( x \)-axis.

(b) the \( y \)-axis.

3. Use the **disk/washer method** to find the volume of the solid obtained by rotating the region bounded by \( y = x^3, y = 8, x = 0 \) about the line \( x = 3 \).

4. Find the area of the region bounded by the curves \( y = x^2 \) and \( y = |x| \).

5. Use the **method of cylindrical shells** to find the volume of the solid obtained by rotating the region bounded by \( y = 4x - x^2 \) and \( y = 0 \) about the line \( x = -1 \).

Weekly Assignment #02

1. Evaluate the following definite or indefinite integrals.

   (a) \( \int \tan x \ln(\cos x) \, dx \).

   (b) \( \int \frac{e^x}{(e^{2x} + 3)^3} \, dx \).

   (c) \[ \int_1^\infty \frac{1}{x^2} \ln x \, dx \] if \( \int_0^1 f(x) \, dx = 3 \).

2. Compute \( F'(x) \).

   (a) \( F(x) = \int_2^{x^2} \ln t \, dt \).

   (b) \( F(x) = \int_{2x}^{x^2} e^{t^2} \, dt \).

Weekly Assignment #03

1. Compute \( \int x \cos x^2 \, dx \).

2. Compute \( \int \sin x \cos^2 x \, dx \).

3. Compute \( \int \cos^2 x \, dx \).

4. Compute \( \int x \sqrt{2-x} \, dx \).

5. Evaluate \( \int \frac{1}{1 + e^x} \, dx \).
6. Evaluate $\int_{0}^{1} (x^2 + 1)e^{-x} \, dx$.

**Weekly Assignment #04**

1. Evaluate $\int x^3 e^x \, dx$.
2. Evaluate $\int x^2 e^{-3x} \, dx$.
3. Evaluate $\int \arcsin x \, dx$.
4. Evaluate $\int \sin^2 x \, dx$ in two different methods.
5. Evaluate $\int \frac{x^3}{(x^2+1)^2} \, dx$ in three different methods.
6. Evaluate $\int (x + 1)^3 \ln 3x \, dx$.
7. Evaluate $\int e^{-x} \sin 4x \, dx$.
8. Evaluate $\int_{0}^{\frac{1}{2}} xe^{2x} \, dx$.
9. Evaluate $\int_{0}^{\frac{\pi}{4}} x \sin 2x \, dx$.

**Weekly Assignment #05**

1. Evaluate $\int_{0}^{\frac{\pi}{4}} x \sin 2x \, dx$.
2. Evaluate $\int \frac{\tan^2 x}{\sec^5 x} \, dx$.
3. Evaluate $\int \cos^n x \, dx$.
4. The axis of a storage tank in the form of a right circular cylinder is horizontal. The radius and length of the tank is 1m and 3m respectively. Determine the volume of fluid in the tank as a function of its depth "d".
5. Evaluate $\int x\sqrt{4+x} \, dx$ by integration by part and substitution.
6. Evaluate $\int \frac{3}{\sqrt{1-x^2}} \, dx$.
7. Evaluate $\int \frac{3x}{\sqrt{1-x^2}} \, dx$.
8. Evaluate $\int \frac{3x^2}{\sqrt{1-x^2}} \, dx$. 
Weekly Assignment #06

1. Evaluate \( \int \sqrt{1 + \sqrt{x}} \, dx \).
2. Evaluate \( \int \frac{1}{(x^2 + 1)^2} \, dx \).
3. Evaluate \( \int \frac{x}{(x^2 + 1)^2} \, dx \).
4. Evaluate \( \int \frac{3x^3 + 4x}{(x^2 + 1)^2} \, dx \).
5. Evaluate \( \int \frac{1 - x + 2x^2 - x^3}{x(x^2 + 1)^2} \, dx \).
6. Show that \( \int \frac{1}{\sqrt{x}} \ln x \, dx \) converges.

Weekly Assignment #07

1. Evaluate \( \int \frac{x}{\sqrt{1 + x}} \, dx \).
2. Evaluate \( \int \frac{\cos^3(e^{-x})}{e^x} \, dx \).
3. Evaluate \( \int \frac{1}{x\sqrt{4x^2 - 1}} \, dx \).
4. Evaluate \( \int \frac{1}{x^4\sqrt{9x^2 - 1}} \, dx \).
5. Evaluate \( \int_{0}^{\infty} e^{-px} \sin qx \, dx \), where \( p \) is positive and \( q \neq 0 \).
6. Evaluate \( \int_{-\pi}^{\pi} x^2 \cos nx \, dx \), where \( n \) is integer.
7. Evaluate \( \int_{0}^{\pi/2} \frac{1}{2 + \cos x} \, dx \).
8. Solve \( x' = 1 - t + x - tx, \ x > 0 \).
9. Solve \( y' + y = \sqrt{x}e^{-x}, \ y(0) = 3 \).
10. A tank contains 50 kg of salt dissolved in 1000 L of water. Brine that contains 10g of salt per litre of water enters the tank at a rate of 10 L per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half of an hour?

11. According to Newton's law of cooling (or heating), the time rate of change of the temperature \( u(t) \) of a body immersed in a medium of constant temperature \( A \) is proportional to the temperature difference \( u - A \). It follows that

\[ u' = -k(u - A), \]

where the proportionality constant \( k \) is positive.

A 4 lb roast, initially at 50\(^{\circ}\) F, is placed in a 375\(^{\circ}\)F oven at 5:00pm. At 6:15pm, the temperature of the roast is 125\(^{\circ}\) F. When will it be ready to serve medium rare (at 150\(^{\circ}\)F)?
Weekly Assignment #08

1. Determine if the following sequences converge or diverge. If the sequence converges determine its limit.
   (a) \( a_n = n^2 e^{-n} \)
   (b) \( \left\{ \frac{1 + n}{2 - 3n} \right\} \)
   (c) \( \{ n \sin \left( \frac{1}{n} \right) \} \)
   (d) \( \{ 1 + (-1)^n \} \)
   (e) \( \left\{ \frac{5n}{\sqrt{n^2 + 4}} \right\} \)
   (f) \( \{ \sqrt{n + 1} - \sqrt{n} \} \)

2. Find the sum: \( \sum_{n=6}^{\infty} \ln \left( \frac{n + 1}{n + 2} \right) \).

3. Find the sum: \( \sum_{n=0}^{99} \frac{1}{n^2 + 6n + 8} \).

4. Let \( a_1 = 8, a_n = 2 + \frac{1}{2} a_{n-1} \), for \( n > 1 \). Show that \( \{ a_n \} \) converges and find its limit.

Weekly Assignment #09

1. Determine whether the sequence converges or diverges.
   \( a_n = \frac{(-3)^n}{n!} \).

2. Determine whether the series converges or diverges.
   \( \sum_{n=1}^{\infty} \cos(n\pi) \).

3. Find the sum of series:
   \( \sum_{n=0}^{\infty} \frac{3^{n+2}}{6^{n-4}} \).

4. Determine whether the series converges or diverges.
   \( \sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}} \).

5. Determine whether the series is convergent or divergent.
   (a) \( \sum_{n=1}^{\infty} \frac{n^3}{n^5 + 3} \)
   (b) \( \sum_{n=1}^{\infty} \frac{3^n}{4^n + 4} \).
(c) \[ \sum_{n=2}^{\infty} \ln(1 - \frac{1}{n^2}) \].

(d) \[ \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 - 1}} \].

(e) \[ \sum_{n=2}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right) \].

**Weekly Assignment #10**

1. Determine whether the series \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \) converges or diverges.

2. Find the values of \( p \) for which the series \( \sum_{n=3}^{\infty} \frac{1}{n\ln(n)[\ln(\ln(n))]^p} \) is convergent.

3. Determine whether the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^4 + 1} \]

4. Determine whether the series \( \sum_{n=1}^{\infty} \frac{1}{n^{1 + \frac{1}{n}}} \) converges or diverges.

5. Find the values of \( p \) for which the series \( \sum_{n=1}^{\infty} \frac{n^p}{2 + n^3} \) is convergent.

6. Determine whether the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^4 + 1} \]

7. Determine whether the series \( \sum_{n=1}^{\infty} \frac{1}{n^{1 + \frac{1}{n}}} \) converges or diverges.

**Weekly Assignment #11**

1. For each of the following, say whether it converges absolutely, converges conditionally or diverges and explain why.

   (a) \[ \sum_{n=1}^{\infty} (-1)^n \frac{n!}{\pi^n} \]

   (b) \[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2 + 1}} \]

   (c) \[ \sum_{n=1}^{\infty} \frac{n + 5}{n\sqrt{n + 3}} \]

   (d) \[ \sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right) \]

2. For each of the following, say whether it converges absolutely, converges conditionally or diverges and explain why.

   (a) \[ \sum_{n=1}^{\infty} \frac{3 + \cos n}{e^n} \]
3. Consider the sequence defined by \( a_n = \frac{(-1)^n + n}{(-1)^n - n}. \) Does this sequence converge and if it does, to what limit.

**Weekly Assignment #12**

1. For each of the following, say whether it converges absolutely, converges conditionally or diverges and explain why.

(a) Determine whether the series \( \sum_{n=1}^{\infty} \frac{n^4}{n!} \) converges or diverges.

(b) Consider the series with terms \( a_n = \frac{n}{2n} \) when \( n \) is odd and \( a_n = \frac{1}{2n} \) when \( n \) is even. Does \( \sum_{n=1}^{\infty} a_n \) converge?

(c) Evaluate the indefinite integral \( \int \frac{t}{1 - t^3} dt. \)

(d) Find the first 3 non-zero terms of a Taylor series expansion of \( f(x) = \sqrt{x} \) centered at 4.

2. Consider the convergent series \( \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n}. \) Without using the exact value of the sum, how many terms of the series do we need to add in order to approximate the sum with an error that is smaller than 0.3? Explain.

**Weekly Assignment #13**

1. Determine whether the series \( \sum_{n=1}^{\infty} e^{-n} (1 + \frac{1}{n})^{2021} \) is convergent or divergent. Use the Limit Comparison Test only. *No credit will be given using other methods.*

2. Use a Taylor polynomial of degree 5 to approximate \( \int_{-1}^{1} \frac{\sin(x)}{x} dx. \)

3. Find the sum of the series.

(a) \( \sum_{n=0}^{\infty} \frac{(-1)^n (n)^{2n}}{6^{2n}(2n)!} \)

(b) \( \sum_{n=0}^{\infty} \frac{3^n}{5^n n!} \)

4. Estimate the error if only the first three terms of Taylor series of \( f(x) = \sqrt{x} \) centered at 4 were used when \( x \) lies in the given interval \( 4 \leq x \leq 4.2. \)

**Weekly Assignment #14**

1. Sketch the graph of \( r = \sin(2\theta). \)

2. Find the area of one petal of the graph defined by the equation \( r = \sin(2\theta). \)

3. Determine the area that lies inside \( r = 3 + 2\sin\theta \) and outside \( r = 2. \)