Power, Property Rights, and the Dynamics of Local Wealth Appropriation

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Abstract

This paper examines the dynamic connections between wealth inequality and political decisions at the local level. We model a jurisdiction consisting of a politically dominant group and a marginalized one. At each date, the dominant group tries to appropriate ownership claims on productive assets. If its claims are legally challenged, the default outcome is determined by a right of possession (ROP) principle. Despite the ROP’s assurance of equal protection, the jurisdiction systematically redistributes property claims toward the dominant group. The jurisdiction appropriates wealth by leveraging common assets - those that generate non-rival consumption flows - using zoning, takings, or NIMBY policies. We examine how this leverage varies across time and depends on whether a common asset produces a public good or a NIMBY (producing public benefits for the dominant group and harms for the marginalized one). Finally, ROPs that prioritize some assets over others can exacerbate inequality.

JEL Codes: C73, D31, D78, H13, P48, R52.

Key Words and Phrases: Property assignment, wealth appropriation, right of possession, private, public, and NIMBY assets, zoning, takings, Dynamic Samuelson condition, asset durability and prioritization.

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1 Introduction

Open societies often face a tension between individual rights and distributive politics. This tension plainly surfaces in local disputes over property rights. Laws on the books safeguard equal protection, limits on property takings, and due process. Yet an expanding empirical literature links policies like exclusionary zoning, NIMBY (“not in my backyard”) restrictions, eminent domain, and takings to increases in inequality within metropolitan areas.

Rothwell and Massey (2009b), Rothwell and Massey (2009a), Rothwell (2011), Prevost (2013), Shertzer et al. (2016a), Trounstine (2016), and Trounstine (2018) show that increased zoning stringency is associated with increased segregation and inequality. Glaeser and Gyourko (2002), Furman (2015) and White (2015) find that these regulations inflate housing prices and values in wealthier areas. Shertzer et al. (2016a) and Mayo (2020) examine targeted NIMBY zoning. They find evidence of targeted NIMBY zoning that places toxic landfills and industrial infrastructure in poorer neighborhoods. Trounstine (2018) finds that the longer a zoning policy exists, the more pronounced the level of segregation.¹

The tension between legal protections and zoning is observed by Illya Somin, a panellist for the U.S. Commission on Civil Rights (Briefing (2014)), who testified:

“Americans of all racial and ethnic backgrounds have suffered from government violations of constitutional property rights. But minority groups have often been disproportionately victimized, sometimes out of racial prejudice and at other times because of their relative political weakness. Minorities are especially likely to be victimized by private to private condemnations that test the limits of the Public Use Clause of the Fifth Amendment, which requires that property can only be condemned for a ‘public use.’ These include takings allegedly justified by the need to alleviate ‘blight’ and promote ‘economic development.’”

Political constraints on zoning may be even weaker than legal ones. A number of studies point to low levels of political competition and turnover at the local level,² suggesting that local elites are not too concerned about losing power. Other studies


² Ferreira and Gyourko (2009), Trounstine (2011), and de Benedictis-Kessner (2017) all document local incumbency advantages. Einstein et al. (2019) shows demographically stable groups, typically older, male, longtime residents, and homeowners, dominate local politics. Lee and Lee (2021) show that
show that local political elites have in fact used zoning to redistribute wealth toward their core supporters (Katznelson (2005) and Trounstine (2018)). “Wherever these elites live, their wealth and connections make them influential forces within local society.” - Wyman (2021).

The present paper formalizes this influence. We propose a theory linking wealth capture and inequality to political choices at the local level. We posit a dynamic game model of a political jurisdiction consisting of two groups, politically dominant “in-group” and a marginalized “out-group.”

At each date $t$, the in-group attempts to reallocate ownership claims on assets. Each asset generates a stream of returns of a particular good. The in-group tries to appropriate wealth using policy instruments such as exclusionary zoning, NIMBY location, takings, and eminent domain. These instruments are modeled by their distributive effects on asset ownership.

In the absence of any constraint on its behavior, the in-group would clearly try to expropriate all private claims of the out-group. The jurisdiction, however, exists within a larger legal framework that offers some protections to the out-group. Individuals can contest changes to their property rights. A contested claim is adjudicated and resolved according to a legal right of possession (ROP) principle. This principle, common to most western democracies, confers a level of protection to the current legal “possessor” of the asset. In the U.S. court system, rights of possession (ROPs) are based on precedent and on state and federal law. They govern eminent domain, compensation of property holders, environmental protection, and so forth. Both the U.S. Constitution’s 5th Amendment and the EU’s Article 1, Protocol 1 regulate state takings of property which, in turn, shape the ROPs within their borders.

Long run political power of the in-group is therefore constrained to some extent by legal protections that ROPs provide to the out-group. We characterize wealth paths of each group in a smooth Markov Perfect equilibrium. In equilibrium the paths are non-stationary. Generally, wealth is redistributed upward. While the absolute level of wealth of both groups may be increasing over time due to economic growth, the share going to the in-group increases.

In the model, the wealth dynamics are determined by two factors: (1) the level of protection offered by the ROP, and (2) the type of assets and policy instruments available to the in-group. An ROP’s level of protection is determined by its durability. Durability describes the extent of an owner’s right to the asset’s growth. At one extreme, a fully durable ROP guarantees the owner the full rights to the asset including the rights to growth in its value. At the other extreme, a status quo ROP guarantees only the right to

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3 The terminology is not new. See Pistor (2019).
the pre-existing claim, but no right to its potential growth. The differences reflect both the level and timing of compensation for takings. An offer to compensate the current owner at roughly the value that prevailed before the attempted taking may satisfy a *status quo* ROP but not a fully durable one. More redistribution is possible when the gap between the actual ROP and the fully durable one is large.

The level of redistribution also depends on which assets and which instruments are available. We highlight the leveraging role of *common assets* for wealth appropriation. More precisely, a complete specification of an individual’s portfolio of ownership claims should include both his private assets and the common assets held by the jurisdiction itself. Private assets generate consumption streams of fully rival (private) goods like housing. Common assets generate consumption streams of non-rival goods. These include public assets which generate public goods (public safety, infrastructure, parks), and NIMBYs which, when located in the out-group’s neighborhood, generate benefits for the in-group and harms to the out-group (landfills, prisons, water treatment plants).

In order to increase its wealth share, the in-group must take either from the private wealth of the out-group or from the common assets. While an ROP constrains the appropriating capacity of the dominant group, the presence of a common asset eases this constraint.

The trade-offs are illustrated in the case of one private and one common asset. As the wealth distribution evolves, the in-group continually adjusts ownership claims to satisfy a *Dynamic Samuelson condition* in which the sum of the two groups’ long run marginal rates of substitution between claims on the common asset and claims on the private asset equals the marginal rate of transformation between the assets.

When the common asset is a public asset (i.e. everyone benefits), the in-group’s private wealth share comes at the expense of the out-group’s. The wealth share of the marginalized group is shown to fall over time, while that of the in-group increases. The wealth share of the public asset can increase or decrease, however, we show that in-group private wealth relative to public wealth share is increasing, and out-group wealth relative to public wealth decreases. In other words, private wealth inequality increases faster than the private-to-public wealth dispersion. The reason has to do with the non-rival aspect of the public asset. To appropriate wealth, the in-group uses the public asset as leverage to “buy off” the out-group by offering mutually beneficial increases in the public asset. This creates slack in the legal constraint and increases the marginal gain from private wealth for in-group. Modest increases in public assets can therefore lead to large increases in out-group appropriation.

However, this leveraging works only if the provision of the public asset does not exclude the out-group. If, for instance, the jurisdiction maintains parks only in the in-group neighborhoods, then the “public” asset is essentially a private one at the group level. In that case, the ability of the in-group to appropriate wealth from the out-group is limited. In fact, in some cases, appropriation of out-group wealth can only occur if
genuinely public assets are leveraged. We characterize the shadow value of leverage and show that it decreases over time as the out-group’s wealth share is depleted. In the stationary limit, leveraging becomes unnecessary.

There are numerous examples of apparent leveraging of this sort. As governor of Alabama, for instance, the well-known segregationist George Wallace increased education funding for public education to both predominantly Black and White colleges to mute objections to segregation (Katsinas (1994)). In another example, after World War II the GI Bill was used by communities as leverage to maintain occupational segregation. Local colleges and trade schools attracted Black veterans who could use funding from the GI Bill. At the same time, Black veterans were deterred from starting businesses due to discriminatory lending practices (Katznelson (2005) and Blakemore (2019)). These practices were also used in mortgage lending to segregate neighborhoods (Wiese (2004)). More recently, the city of San Francisco combines generous public funding with exclusionary zoning that, again, resembles the leveraging strategy modeled here (Lowrey (2022)).

Similar examples are found when the common asset is a NIMBY. If the NIMBY harm to the out-group is moderate and its benefit to the in-group is not too high, the in-group uses the threat to expand the NIMBY in the poor neighborhood to appropriate the out-group’s wealth. While the out-group’s private wealth may be “allowed” to grow to compensate for the NIMBY harm, the growth is lower than for the in-group and so inequality increases over time. NIMBY decisions by the Texas Commission on Environmental Quality, for instance, were shown to disproportionately pollute minority neighborhoods in the Dallas area (Fears (2020) and Mayo (2020)). The dispute over potential expansion of Shingle Mountain, the local landfill in Dallas, lowered property values nearby.4

We also show that if the NIMBY harm is great enough, then leveraging is limited. In that case, wealth inequality can actually decrease over time.

Finally, we study the effects of prioritization, i.e., ROPs in which some assets are more durable than others. Effects of prioritization in debt and bankruptcy is a common topic (Pistor (2019)). Less is known about distributional effects of prioritization in property appropriation. Right of possession rules are highly non-uniform across states and nations. We show that prioritization can exacerbate inequality. ROPs in which public (private) assets are more durable will increase the rate of wealth appropriation from the out-group when public (private) assets are valued more highly than private ones. Hence, the prioritizing of certain assets allows for greater wealth appropriation by the in-group.

Overall, the model emphasizes two important features of local, as opposed to national, inequality. First, increases in inequality in the model are driven by political and

4These examples are described in more detail in Section 4.2.
legal, not technological, factors. This is consistent with evidence that local inequality is
driven by neighborhood effects rather than by differential impacts of technology (Chetty
and Hendren (2018a,b)). Second, while not discriminatory, ROPs do not compensate
for asymmetries in political power. Privileged groups have an array of policies that
redistribute wealth “upward.” The legal constraints slow but do not halt the rate of
appropriation.

The paper’s organization follows a standard layout. Section 2 presents the model.
Section 3 solves for the equilibrium. It establishes the key intermediate result - the
Dynamic Samuelson condition - used in all subsequent results. Section 4 examines
the canonical two-asset case. Then Section 5 extends the canonical model to allow
for prioritization of assets. Section 6 relates the model to other models and the wider
discussion of local inequality in the literature. The last section is the Appendix.

2 The Model

The patterns in the previous section suggest that seemingly obscure political decisions
on zoning translate into large redistributions of wealth. This section describes a model
with this feature. The section lays out the group demographics, the asset and ownership
structure, and then the legal environment.

2.1 In-Group and Out-Group

Consider an ongoing local polity with two groups, a politically dominant “in-group” and
a marginalized “out-group.” The two groups are assumed to be internally homogeneous,
and for simplicity all decisions will take place at the group rather than individual level.
Denote the in-group by $R$ (the “rulers”), and out-group by $P$ (the “powerless”).

We assume that in-group $R$ permanently holds power, meaning that at all times
local policies reflect the in-group’s preferences subject to certain legal constraints (to
be described shortly). We take the permanent dominance of group $R$ as given and do
not model the reasons. It could be due to demographics that give $R$ an ongoing size
advantage. Alternatively, $R$ could be a numerical minority but successfully suppresses
the votes or biases the rules in its favor. Whatever the reason, there is evidence of level
levels of political competition and turnover in the U.S. at the local level.\footnote{Footnote6
references a few of these studies.}
2.2 Ownership of Assets

The time horizon is infinite. At each date $t$, the in-group chooses policies that have the effect of assigning ownership to wealth-generating assets. Each asset generates a stream of returns of a particular type of good. There are $s$ such assets, all denominated in a common unit of account. An ownership claim $a_{jkt}$ describes a property right (in the common unit of account) of a group $j = R, P$ member to the flow of returns from asset $k$ in period $t$.

An ownership assignment is allocation of claims across groups and across assets. Formally, an ownership assignment at date $t$ is a collection

$$a_t = (a_{R1t}, a_{P1t}, \ldots, a_{Rst}, a_{P1t}, \ldots, a_{Pst}).$$

Ownership assignments differ across groups but for simplicity are assumed the same within each group.

For simplicity, one unit of a claim on asset $k$ returns one consumption unit of $k$ each period. Payoffs can then be defined directly on ownership claims. Let $u_j(a_t)$ denote the flow payoff of assignment $a_t$ to a member of group $j$ at $t$. The payoff $u_j$ is assumed to be smooth, concave, and for each $k$ monotonic (either increasing or decreasing everywhere) in $a_{jk}$. We also assume there exists smooth real valued functions $f_j : \mathbb{R}_+ \to \mathbb{R}_+$ and $g_j : \mathbb{R}_+^{2s} \to \mathbb{R}_+$ such that for any $\epsilon > 0$, $\frac{\partial u_j(\epsilon a)}{\partial a_k} = f_j(\epsilon) \frac{g_j(a)}{\partial a_k}$. This homotheticity assumption sets aside income effects to focus on political mechanisms for determining property rights. The long run payoff to an individual of group $j$ is $\sum_{t=0}^{\infty} \delta^t u_j(a_t)$ where $\delta$ is the common discount factor.

Some assets generate a stream of consumption that is exclusive or rival. We refer to these as private assets. Housing is a standard example. Others assets generate non-rival consumption streams which we refer to as common assets. Among common assets, some generate non-rival consumption streams that benefit everyone, for instance, public safety, infrastructure, a water table, or clean air. Other common assets generate benefits for some individuals while harming others. Examples are land-fills, waste storage, or power grids. Common assets that benefit everyone will be referred to as public assets since they essentially produce public goods. Common assets that harm a subgroup will be called NIMBYs. As the name suggests, the harm from NIMBYs is typically due to close proximity to the location of the asset.

Let $\bar{a}_{kt}$ denote the total claim on asset $k$ at date $t$. Depending on whether $k$ is a private or common asset, it is not necessarily true that $\bar{a}_{kt} = a_{Rkt} + a_{Pkt}$. Formally, the three categories of assets are defined below:

- **Private Assets.** Asset $k$ is a private asset if $\bar{a}_k = a_{Rk} + a_{Pk}$ and $\frac{\partial u_j(a)}{\partial a_k} > 0$ and $\frac{\partial u_j}{\partial a_{j'k}} = 0$ for $j, j' = R, P$ with $j \neq j'$. A private asset is one that is fully rival and $j$’s claim $a_{jk}$ only affects $j$’s payoff.
• **Public Assets.** Asset $k$ is a public asset if $\bar{a}_k = a_{Rk} = a_{Pk}$ and $\frac{\partial u_j}{\partial \bar{a}_k} > 0$ for both $j = R, P$. Ownership and consumption from the public asset is non-exclusive, non-rival, and benefits both groups.

• **NIMBY Assets.** Asset $k$ is a NIMBY ("Not in my back yard") asset if $\bar{a}_{kt} = a_{Rkt} = a_{Pkt}$ and $\frac{\partial u_P}{\partial \bar{a}_k} < 0$ and $\frac{\partial u_R}{\partial \bar{a}_k} > 0$. Ownership and consumption from the NIMBY is non-rival, but benefits the in-group at the expense of the out-group.\(^6\)

We assume that ownership of asset $k$ is positively valued by at least one of the two groups, i.e., $\frac{\partial u_j}{\partial a_{jk}} > 0$ for one or both $j = R, P$ so that all assets fall in one of these three groups. From here on out, we assume at least one private asset and at least one common asset.

We assume that a unit of asset period $t$ can be costlessly converted to another in $t+1$. All assets have growth potential of $\gamma > 1$, meaning that if there is no redistribution, all claims will grow at identical rate $\gamma$. Because the allocation across assets is politically determined, an asset’s realized growth rate can differ from its potential rate $\gamma$. Each period, resource constraints on total claims depend on the total claims inherited from the previous period. Thus, to be feasible, an ownership assignment $a_t$ must satisfy

$$\sum_k \bar{a}_{kt} \leq \sum_k \gamma \bar{a}_{kt-1}. \quad (1)$$

Equation (1) merges a time transition rule and a cross-sectional resource constraint. It assumes a production technology in which claims are convertible one-for-one across different asset classes. This is consistent with an interpretation of asset claims all arising from a uniformly measured resource such land, energy, or gigabytes of RAM that can be allocated for different uses. There are no explicit prices or markets; claims are chosen by political decision makers.

### 2.3 Right of Possession Principles

In some cases a change in the assignment $a_t$ from the previous period’s one, $a_{t-1}$, maps directly to a commonly observed policy. Using the feasibility constraint one, $a_{t-1}$, maps to a commonly observed policy. Using the feasibility constraint Equation (1), if $k$ is the single public asset and $\bar{a}_{kt} < \gamma \bar{a}_{kt-1}$ then $a_t$ privatizes some of the public asset $k$. Whereas, if $k$ is a single public asset and $\bar{a}_{kt} > \gamma \bar{a}_{kt-1}$ then $a_t$ includes an assertion of eminent domain that moves private assets into the public one.

\(^6\)NIMBYs are often thought of as public goods for the then community-at-large but public “bads” for those the live nearby. Defining the NIMBY in terms of payoffs is shorthand for a location of the asset in the out-group’s neighborhood when the two groups are geographically separated. We omit the case in which the NIMBY benefits $P$ and harms $R$ (e.g., the NIMBY is in $R$’s neighborhood).
Suppose instead that $k$ is a private asset and $a_{Rkt} - \gamma a_{Rkt-1} = \gamma a_{Pkt-1} - a_{Pkt} > 0$. Then $a_t$ includes a direct private transfer of wealth from out-group $P$ to in-group $R$. This can come about from, say, a zoning restriction that excludes the out-group. A more direct method is a private taking: the locality essentially transfers ownership of an asset from one private citizen or firm to another. The courts adjudicate the level of compensation to the original owner.\footnote{In the lead up to what eventually became a famous U.S. Supreme Court case, \textit{Kelo v. City of New London}, the city of New London sought in 2002 to appropriate the private property of owner Susette Kelo. It sought her property for a private development project by the New London Development Corporation. In court the city successfully argued that this form of private taking falls under eminent domain - since the private development would generate public benefits for the city.}

In practice, the assignment $a_t$ is a complex result of many local policies, combining eminent domain, NIMBY expansion, and density restrictions. Rather than model the details of these policies, we focus on their distributive effect. Thus, the assignment $a_t$ is assumed to be chosen directly by $R$, subject to certain constraints.

At each date, the in-group $R$ attempts to impose an ownership assignment $a_t$. Group $R$’s decision is summarized by an ownership assignment rule, a Markov strategy $\pi(a_{t-1}) = a_t$ that transforms the prior assignment into a current one and satisfies the feasibility constraint (1). The dynamic payoff to a group $j = R, P$ when assignment rule $\pi$ is used is expressed recursively as

$$V_j(a_{t-1}; \pi) \equiv u_j(\pi(a_{t-1})) + \delta V_j(\pi(a_{t-1}); \pi)$$ \hspace{1cm} (2)

The state variable in the recursive payoff is the prior claim $a_{t-1}$.

Whether a proposed assignment $a_t$ is implemented depends on whether the legal process constrains the dominant group’s rapaciousness. Such constraints exist. Here, they take the form of a due process clause that allows any individual to contest any re-assignment of the property claims. In turn, a contested claim results in an assignment $a^\circ_t$ determined by a default rule, denoted by $\pi^\circ$, governed by a right of possession (ROP) principle. The ROP confers a baseline level of protection of property rights to those in possession of an asset.\footnote{Since “possession” of a NIMBY harms the possessor, the acronym ROP could more accurately stand for Responsibility of Possession.}

So, if $a_t$ is contested, the ROP results in the claim $a^\circ_t \equiv \pi^\circ(a_{t-1})$. Consequently, a proposed assignment $\pi(a_{t-1})$ from assignment rule $\pi$ is implemented if only if

$$V_P(a_{t-1}; \pi) \geq u_P(a^\circ_t) + \delta V_P(a^\circ_t; \pi).$$ \hspace{1cm} (3)

In (3), the value to the marginalized group must be no less than what it can get by contesting the claim under the ROP rule $\pi^\circ$.

For tractability, restrict attention to ROP rules of the form

$$\pi_{jk}^\circ(a_{t-1}) = \beta_k \gamma a_{jkt-1}$$ \hspace{1cm} (4)
for each asset $k$ where $\beta_k \in [1/\gamma, 1]$. The parameter $\beta_k$ is associated with the ROP to a claim on asset $k$. An owner who contests an assignment is guaranteed some portion $\beta_k$ of the value of the asset $k$ at the beginning of the period. ROPs are ostensibly nondiscriminatory; they do not vary by group identity.

When $\pi^\circ$ is an ROP default rule, the constraint (3) is referred to as the ROP constraint. By varying the parameter $\beta_k$ in the interval $[1/\gamma, 1]$, one varies the strength or durability of the ROP. At the upper bound $\beta_k = 1$, the default claim of group $j$ member is $\pi^\circ_{jk}(a_{t-1}) = \gamma a_{jkt-1}$. We refer to the ROP for asset $k$ when $\beta_k = 1$ as fully durable. Each individual is guaranteed a claim on the asset at its full growth potential.

At the lower bound $\beta_k = 1/\gamma$, the default claim of group $j$ member is $\pi^\circ_{jk}(a_{t-1}) = a_{jkt-1}$. We refer to the ROP for asset $k$ when $\beta_k = 1/\gamma$ as a Status Quo Right of Possession. Under a status quo ROP, an individual retains only the rights to his pre-existing claim, and no claim to any of the asset’s subsequent growth.

In general, the more durable (larger) is the right of possession $\beta_k$, the better the outcome for the out-group when asset $k$ is private or public. However, when the asset is a NIMBY, a more durable right of possession is a millstone. More durable NIMBYs force the out-group to absorb a larger share of harmful returns under the default rule. In addition, since durability can differ across assets, an asset with a higher $\beta_k$ than another is said to have higher priority under contested appropriations.⁹

### 3 The Political Game

The political game between the two groups is represented by a sequentially constrained dynamic decision problem. We will refer to $\pi^*$ as an equilibrium if it solves at each date $t$ and each pre-existing claim $a_{t-1}$,

$$\max_{\pi} V_R(a_{t-1}; \pi)$$

subject to feasibility constraint (1) and the ROP constraint (3).

Framed in this way, the model is described simply as follows. The jurisdiction’s in-group has decision authority over property rights but must abide by the ROP constraint imposed by the courts or by a higher political authority. The in-group $R$ will manipulate property rights each period subject to constraint (1) and (3). Since the legal constraints

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⁹The terminology is more or less consistent with a classification developed by Pistor (2019) that describes four key legal attributes of capital: priority, durability, convertibility, and universality. Priority “ranks competing claims” to possession of an asset. Durability “extends priority claims” to future periods. Universality enlarges the jurisdiction over the right of possession. Convertibility allows for freer transfer or sale of these rights. In our model, Universality is implicit. Convertibility is explicitly assumed in the resource constraint (1).
on the jurisdiction depend on the prevailing assignment, each successive implementation of a new assignment alters future constraints. The properties of the equilibrium are analyzed in the remainder of this Section.

3.1 Recursive Substitution in the ROP constraint

Recursive substitution can be used to simplify the ROP constraint (3). Combining the recursive payoff (2) with ROP constraint (3), the ROP constraint (3) becomes

\[ V_P(a_{t-1}; \pi^*) \geq u_P(\pi^o(a_{t-1})) + \delta V_P(\pi^o(a_{t-1}); \pi^*) \]  

(6)

By iterating on the default rule \( \pi^o \), define

\[ \pi^o_t(a_0) \equiv \pi^o(\pi^o(\pi^o(\cdots \pi^o(a_0)))) \]  

(7)

and

\[ u^o_t(a_0) \equiv u_P(\pi^o_t(a_0)) \]  

(8)

is the flow payoff to \( P \), iterated \( t \) periods forward, in the default rule \( \pi^o \).

Since the ROP constraint is satisfied with equality in all future periods, iterate forward the right-hand side of ROP constraint (6) to obtain

\[ V_P(a_{t-1}; \pi^*) \geq V^o_P(a_{t-1}) \equiv \sum_{\tau=t}^{\infty} \delta^{\tau-t} u^o_P(\pi^o_{\tau-t}(a_{t-1})) \]  

(9)

The continuation payoff \( V^o_P(a_{t-1}) \) is exogenously determined by the default rule applied to all periods. Substituting \( V^o_P(a_{t-1}) \) in for \( P \)'s equilibrium continuation payoff, the ROP constraint in period \( t \) becomes

\[ u_P(\pi^*(a_{t-1})) + \delta V^o_P(\pi^*(a_{t-1})) \geq V^o_P(a_{t-1}) \]  

(10)

Equation (10) will be the form of the ROP constraint used in the remainder of the paper. Under recursive substitution the assignment \( \pi^* \) only enters the left-hand side of this equality. It enters the continuation value \( V^o_P \) through its direct effect on next period’s state \( a_t = \pi^*_P(a_{t-1}) \). The continuation \( V^o_P \) is evaluated at default assignment \( \pi^o \) in all future periods.
3.2 Bellman Saddle Problem and Equilibrium

Given the simplified expression of the ROP constraint in (10), the Bellman equation becomes a recursive Lagrangian saddle problem,

\[ V_R(a_{t-1}; \pi^*) = \max_{a_t} \left\{ u_R(a_t) + \delta V_R(a_t; \pi^*) + \varphi_t (u_P(a_t) + \delta V_P(a_t) - V_P^\circ(a_{t-1}) \right\} \]

\[ + \mu_t \left( \sum_k (\gamma \bar{a}_{kt-1} - \bar{a}_{kt}) \right) \]

(11)

where \( \varphi_t \) is the multiplier on the ROP constraint and \( \mu_t \) the multiplier on the feasibility constraint at date \( t \).

An interior solution is characterized by a system Euler equations associated with (11). Consider a claim \( a_{jkt} \) by group \( j \) for asset \( k \) in date \( t \). The interior first order condition for this particular assignment is

\[ \frac{\partial u_R}{\partial a_{jkt}} + \varphi_t \frac{\partial u_P}{\partial a_{jkt}} - \mu_t = -\delta \left[ \frac{\partial V_R}{\partial a_{jkt}} + \varphi_t \frac{\partial V_P^\circ}{\partial a_{jkt}} \right] \]

(12)

The state gradient of the value function \( V_R \) is

\[ \frac{\partial V_R}{\partial a_{jkt}} = \gamma \mu_{t+1} - \varphi_{t+1} \frac{\partial V_P^\circ}{\partial a_{jkt}} \]

(13)

Substituting (13) into (12), we obtain the Euler equations

\[ \frac{\partial u_R}{\partial a_{jkt}} + \varphi_t \frac{\partial u_P}{\partial a_{jkt}} - \mu_t = -\delta \left( \varphi_t - \varphi_{t+1} \right) \frac{\partial V_P^\circ}{\partial a_{jkt}} + \gamma \mu_{t+1} \]

(14)

each \( j = R, P \) and \( k = 1, \ldots, s \). Hence, from Equations (7) and (9),

\[ \frac{\partial V_P^\circ}{\partial a_{jkt}} = \sum_{\tau=1}^\infty (\delta \gamma \beta_k)^\tau \frac{\partial u_P^\circ(\pi_\tau)}{\partial a_{jkt}} \]

(15)

where \( \pi^\circ(\pi_\tau) \) is defined by (7). Substituting (15) into (14), the Euler equation for each claim \( a_{jkt} \) is reduce to

\[ \frac{\partial u_R}{\partial a_{jkt}} + \varphi_t \frac{\partial u_P}{\partial a_{jkt}} - \mu_t = -\left( \varphi_t - \varphi_{t+1} \right) \sum_{\tau=1}^\infty (\delta \gamma \beta_k)^\tau \frac{\partial u_P^\circ(\pi_\tau)}{\partial a_{jkt}} + \delta \gamma \mu_{t+1} \]

(16)
When \( k \) is a public asset by both groups, then from Eq. (12), the left-hand side of Equation (16) is negative. Intuitively, in a static world \( R \) chooses asset \( k \) so that its constrained marginal benefit \( \frac{\partial u_R}{\partial a_{jkt}} + \varphi_t \frac{\partial u_P}{\partial a_{jkt}} \) equals marginal shadow cost \( \mu_t \) of the feasibility constraint. The static first order condition holds in a steady state where \( \varphi_t = \varphi_{t+1} \). In the dynamic world, \( R \) increases the ownership claims on a public asset \( k \) beyond the static solution in order to relax the ROP constraint in \( t+1 \). The reverse is true for a NIMBY asset. After some point, there is a tradeoff between date \( t \) and \( t+1 \) ROP constraints, and this tradeoff itself varies over time.

### 3.3 The Dynamic Samuelson Condition

Combining Euler equations in (16) across any two assets, the trade offs are summarized by a Dynamic Samuelson condition, a generalization of the classic Samuelson condition.

Consider any pair of assets, one private and one common. The common asset can be either public or NIMBY. Let \( k = 1 \) denote this common asset. Consumption of the common asset depends only on the joint, aggregate claim \( \bar{a}_1 \). Let \( k = 2 \) denote any private asset. For this asset, there are distinct ownership claims \( a_{R2} \) and \( a_{P2} \) for each group.

**Theorem 1** Let \( \pi^* \) be any interior equilibrium.

For any initial claim \( a_0 \), the equilibrium path \( \{a^*_t\} \) of \( \pi^* \) satisfies a Dynamic Samuelson condition of the form

\[
\frac{\partial u_R(a^*_t)}{\partial \bar{a}_{1t}} + \frac{\partial u_P(a^*_t)}{\partial \bar{a}_{1t}} + \delta (1 - \varphi_{t+1}/\varphi_t) \sum_{\tau=1}^{\infty} (\delta \gamma \beta_1)^\tau \frac{\partial u_{R\tau}(a^*_t)}{\partial \bar{a}_{1t}} + \delta (1 - \varphi_{t+1}/\varphi_t) \sum_{\tau=1}^{\infty} (\delta \gamma \beta_2)^\tau \frac{\partial u_{P\tau}(a^*_t)}{\partial \bar{a}_{2t}} = 1 \tag{17}
\]

for each date \( t \).

Suppose further that \( \beta_1 = \beta_2 \) (no asset has priority). Then the Dynamic Samuelson condition reduces to the standard, intra-temporal Samuelson condition

\[
\frac{\partial u_R(a^*_t)}{\partial \bar{a}_{1t}} + \frac{\partial u_P(a^*_t)}{\partial \bar{a}_{1t}} + \frac{\partial u_{R}(a^*_t)}{\partial a_{R2t}} + \frac{\partial u_{P}(a^*_t)}{\partial a_{P2t}} = 1 \tag{18}
\]

The proof is in the Appendix. Equation (17) is a dynamic generalization of the classic Samuelson condition applied to asset ownership. The condition is used here as an intermediate step for later characterizations of wealth paths.

To understand what it means, recall that the classic (static) Samuelson condition equates the sum across individuals of marginal rates of substitution (MRS) between private and public consumption with the marginal rate of transformation. Like its static
counterpart, the Dynamic Samuelson condition characterizes a constrained, socially optimal tradeoff between private and common ownership. In this case, however, the tradeoffs are evaluated using long run payoffs.\footnote{A number of papers generalize the Samuelson condition in other dimensions including matching models (Michaillat and Saez (2015)), unemployment frictions (Michaillat and Saez (2018)), and distortionary taxation (Batina (1990) and Kaplow (1996)). We are not aware of dynamic Samuelson conditions applied to property rights elsewhere in the literature.}

In the model, the within-period marginal rate of transformation is equal to one. The gap between Equation (17) and the static Samuelson condition shows up in the out-group’s MRS, which depends both on the exogenously determined payoff \( u_\tau^\tau \) which was defined in (8), and the shadow prices \( \varphi_t, \varphi_{t+1} \). In the absence of a ROP constraint, \( \varphi_t = 0 \) in which case, the Dynamic Samuelson condition collapses to the static Samuelson condition in which MRS in group \( R \) equals 1. This is the global optimum from the group’s point of view. In a steady state, \( \varphi_t = \varphi_{t+1} \) and, once again, Dynamic Samuelson condition collapses to the static Samuelson condition.

The Dynamic Samuelson condition also reveals a key difference between political control of stocks versus flows. Unlike for instance tax flows, changes in asset ownership alter the tangible status quo in the ROP rule. The result is a non-stationary path of shadow costs \( \varphi_t \) of the ROP constraint. These shadow costs are distortions along the transition path that may last the entire time horizon under standard payoff functions like logs. Current assignments have cumulative stock effects on future constraints, and these effects diminish over time as the dynamic path approaches a steady state.

Yet, even far away from the steady state, the shadow cost only shows up in the Samuelson condition when legal asymmetries exist across assets. As Equation (18) shows, when ROPs are not prioritized, the homothetic utility assumption implies that the Dynamic Samuelson condition collapses to the static Samuelson condition. On the other hand, prioritization of some assets over others will typically distort the assignment away from the classic Samuelson condition.

Generally, when \( P \)’s private assets have lower priority, group \( R \) has more leverage. The most beneficial configurations are those that reduce \( P \)’s MRS, ceteris paribus. Clearly, the ideal case from group \( R \)’s point of view is \( \varphi_t = 0 \) in which case \( P \)’s MRS vanishes.\footnote{This is more easily seen from the Euler equations than from the Samuelson condition.} Holdings of low priority assets, those with relatively lower \( \beta_k \), reduce \( P \)’s MRS. These assets reduce the cost of takings and are thus easier to appropriate. Later on, Section 5 shows how prioritization can exacerbate wealth inequality and redistribution. For now, Section 4 specializes the model to the case with two assets and no prioritization (the \( \beta_k \) are all the same). In this case any changes in inequality are entirely driven by the political power of group \( R \) to implement upward redistribution. This Section will show how public or NIMBY assets facilitates this.
4 The Canonical Model

This Section focuses on a canonical case of two assets. One asset is private, the other is a common asset - either public or NIMBY. The two asset model focuses on the trade off between private and common claims of ownership. Let $k = 1$ denote the common asset, and $k = 2$ the private asset. Initially, assume $\beta_1 = \beta_2 \equiv \beta < 1$. That is, ROPs are not fully durable, and neither asset has priority. We later generalize the assumptions to allow for different priority rights.

By Theorem 1, the Dynamic Samuelson condition coincides with the static Samuelson condition in (18). This produces a more tractable model since Lagrangian multipliers $\varphi_t, \varphi_{t+1}$ do not appear in (18). Recall that for public assets, payoffs only depend on the aggregate wealth $\bar{a}_1$. An assignment can therefore be expressed as the vector $a = (\bar{a}_1, a_{R2}, a_{P2})$.

4.1 Equilibrium Path of Ownership Claims and Wealth Shares

This Section develops qualitative features of the equilibrium time path $\{a^*_t\}$. Because claims are fully convertible at unit value, we refer to claims and wealth interchangeably. The shares of total wealth at date $t$ for each of the three assets are denoted by

$$Q^*_1t \equiv \frac{\bar{a}^*_t}{a^*_{R2t} + a^*_{P2t} + \bar{a}^*_t}, \quad Q^*_Pt \equiv \frac{a^*_P2t}{a^*_{R2t} + a^*_{P2t} + \bar{a}^*_t}, \quad Q^*_Rt \equiv \frac{a^*_R2t}{a^*_{R2t} + a^*_{P2t} + \bar{a}^*_t}.$$

Additionally, the share of out-group $P$’s private claim out of the total private claim is

$$S^*_Pt \equiv \frac{a^*_P2t}{a^*_{R2t} + a^*_{P2t}}$$

(and the in-group’s private wealth share is $S^*_Rt = 1 - S^*_Pt$). Notice that $S^*_Pt = \frac{1}{1 + Q^*_Rt/Q^*_Pt}$ so that increases or decreases in private shares match increases or decreases in wealth ratios.

Proposition 1 Suppose the ROP rule $\pi^*$ satisfies $1/\gamma < \beta < 1$. Let $\pi^*$ be an equilibrium with path $\{a^*_t\}$. Then the following hold for all $t > 1$.

(i) Suppose the common asset ($k = 1$) is a public asset. Then the out-group $P$’s private-to-public wealth ratio $Q^*_P2t/Q^*_1t$ is decreasing over time while the in-group $R$’s private-to-public wealth ratio $Q^*_R2t/Q^*_1t$ is increasing. Group $R$’s private-to-private wealth ratio $Q^*_R2t/Q^*_P2t$ relative to group $P$ is increasing. At each date $t$, $a^*_P2t < \beta \gamma a^*_P2t-1$. 

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(ii) Suppose the common asset is a NIMBY asset. Then $P$’s private share $Q_{Pt}^*$ of total wealth is decreasing over time whenever the share $Q_{1t}^*$ of NIMBY wealth is increasing, and at each date $t$, $a^*_{P2t} > \beta \gamma a^*_{P2t-1}$ and $\bar{a}_{1t} > \beta \gamma \bar{a}_{1t-1}$.

The Proposition describes trajectories of wealth shares over time. When the common asset is a public asset both total wealth share $Q_{Pt}^*$ and private wealth share $S_{Pt}^*$ of out-group $P$ decline. The group’s claim lies below its default value $\beta \gamma a^*_{2t-1}$ at each date. However, because $\beta \gamma > 1$ it is still possible that $P$’s level of private wealth increases. When the common asset is a NIMBY, $P$’s share of wealth moves either up or down depending on benefits of the NIMBY to the in-group and costs to the out-group.

4.2 The “Leveraging” Logic of Proposition 1

To better understand the trade offs facing the in-group, consider Part (i). Clearly, when $\beta < 1$ the out-group loses some of the potential growth in its assets if the ownership assignment is contested. This, in turn, relaxes the resource constraint off-path. Hence, some wealth appropriation can occur even if the only policy instrument is a private taking. In that case the out-group receives its default claim which grows over time.

Yet, with the addition of policy instruments that involve the public asset, the decline in group $P$’s private wealth share exceeds the decline that would have resulted under the ROP. In fact, if the ROP’s durability is low, $P$’s wealth level can decline in absolute terms. This would not be possible if a private taking is the only policy instrument.

This “excess” wealth appropriation is due to the in-group’s political power to leverage the public asset. Because the public asset is valued by both groups, an increase in the claim on the public asset makes group $R$ better off and, because it is also valued by group $P$, it further relaxes the ROP constraint. Applying the homotheticity assumption, the Samuelson condition that holds in equilibrium at $t - 1$, must also hold under the ROP rule $\pi^*$ at date $t$. Formally,

$$\frac{\partial u_R(\beta \gamma a_{1t-1}^*)}{\partial \bar{a}_1} + \frac{\partial u_P(\beta \gamma a_{1t-1}^*)}{\partial a_{R2}} + \frac{\partial u_P(a_{2t-1}^*)}{\partial a_{R2}} + \frac{\partial u_P(a_{2t-1}^*)}{\partial a_{P2}} = 1.$$  

(19)

Now consider the locust of all assignments $(\bar{a}_{1t}, a_{R2t}, a_{P2t})$ that satisfy the Samuelson condition. Moving along this locus from the default profile $\beta \gamma a_{1t-1}^*$, group $R$ will optimally (from its point of view) reduce the slack in the resource constraint. How will it do so? Since the Samuelson condition was satisfied in the prior year, the in-group cannot increase both groups’ private claims and still satisfy the Samuelson condition in the current period. Hence, to satisfy the Samuelson condition, the in-group will instead appropriate some of the growth in $P$’s private wealth and then offset the relative loss by offering just enough of an increased claim on the public asset to satisfy the ROP.
constraint. Group $R$ offers more of the mutually beneficial public asset (for which there is no group conflict) in exchange for a disproportionate claim on the private asset.

A number of historical examples appear to follow this leveraging logic. One, mentioned in the introduction, is described by Katsinas (1994) who examines the education policies and politics of the famous segregationist, George Wallace. As governor of Alabama, Wallace proposed a tax hike to fund construction of two-year colleges. The funding benefited both White and Black Alabamans while segregation was strictly, and often violently, enforced. Katsinas shows that the timing of Wallace’s education policy was not accidental. Wallace conjoined two policies, racial segregation and increased funding for public education, to maintain political support and blunt legal challenges.

In another example, Katznelson (2005) and Blakemore (2019) describe local communities and lenders use of the GI Bill after World War II. The GI Bill, passed in the U.S. shortly after the war, included a number of benefits ostensibly available to all veterans who served in the war. These included funding of college education and access to business loans for GIs. Communities needing workers could make use of subsidies in the GI Bill for attending local colleges and trade schools. On the back end, however, veterans were denied business loans through discriminatory lending practices. Thus the GI Bill could be leveraged to attract workers while maintaining occupational segregation. Moreover, the leveraging didn’t need to be substantial. In an era before fair housing and anti-discrimination laws, Black GIs had few legal options. In the terminology of the paper, educational capital of Black veterans had low durability in the right of possession.

As for Part (ii) of the Proposition, when the asset is a NIMBY the wealth patterns are more difficult to pin down because the nature of the “leverage” with a NIMBY asset is complicated. To expropriate the private wealth of $P$, group $R$ must offer some reduction, relative to the maximum growth in the NIMBY. Yet, unlike the public good case, reductions in the NIMBY are detrimental to the dominant group. So whether $P$’s wealth share decreases depends on which margin $R$ can best exploit. This depends on how both groups value private wealth relative to the NIMBY. The parametric model in the next section will make clear that when the NIMBY is more important to both groups relative to private wealth, then the dominant group has the leverage to appropriate private wealth but is unwilling to use it. When the NIMBY is relatively less important, the dominant group has a bit less leverage but is more willing to use what it has. The dominant group has the most leverage when its value of the NIMBY (relative to private wealth) is small and the cost to the out-group is large.

Mayo (2020) and Fears (2020) describe apparent NIMBY leveraging in studies of the local zoning in Dallas, Texas. Mayo (2020) estimates the pollution distribution by zip codes in Dallas by using aggregated pollution permits from the Texas Commission on Environmental Quality (TCEQ). The study documents the disparity in the locations of toxic waste sites between white and minority neighborhoods in Dallas. TCEQ policies disproportionately pollute minority neighborhoods. Periodic expansions of Dallas land-
fills like Shingle Mountain (Fears (2020)) correspond to lower property values nearby. Our model gives one explanation of TCEQ policies and resulting co-movements both wealth and NIMBY shares in a city with the 8th highest level in inequality in the U.S.

4.3 Parametric Model

This Section demonstrates the leveraging effects at work in a parametric model. The model specification is broad enough to include public and NIMBY assets as special cases.

Let payoffs for both groups be given by

\[ u_R = \rho \log(\bar{a}_{1t}) + (1 - \rho) \log a_{R2t} \quad \text{and} \quad u_P = (\rho - \sigma) \log(\bar{a}_{1t}) + (1 - \rho) \log a_{P2t}. \]  \hspace{1cm} (20)

In (20), the parameter \( \rho \) is a payoff weight common to both groups. The parameter \( \sigma \) represents the additional cost (or benefit when \( \sigma < 0 \)) to group \( P \) of consuming the common asset. When \( \rho > \sigma \) the benefit of the common asset outweighs the cost to \( P \), and so the asset is a public asset. When \( \rho < \sigma \) then the cost exceeds the benefit for \( P \) and the asset is a NIMBY.

Note that if \( \sigma < 1 \) then the common asset could be either a public (\( \sigma < \rho \)) or NIMBY asset (\( \sigma > \rho \)). When \( \sigma > 1 \) then the common asset is necessarily a NIMBY. When \( \rho < \sigma < 1 \) then the asset is a NIMBY but imposes relatively modest harm on \( P \) relative to its value of private wealth. Assume \( \sigma < 2\rho \) to guarantee concavity of the joint preference across the two groups.

The Samuelson condition is

\[ \left( \frac{\rho}{1 - \rho} \right) \frac{a_{R2t}}{\bar{a}_{1t}} + \left( \frac{\rho - \sigma}{1 - \rho} \right) \frac{a_{P2t}}{\bar{a}_{1t}} = 1 \]  \hspace{1cm} (21)

Combining the resource constraint \( \bar{a}_{1t} = \sum_k \bar{a}_{k,t-1} - a_{R2t} - a_{P2t} \) with the Samuelson condition (21) we obtain the system of equations

\[ \bar{a}_{1t} + \sigma a_{2P} = \rho \gamma \sum_k \bar{a}_{k,t-1}, \quad \text{and} \]

\[ a_{R2t} + (1 - \sigma)a_{P2t} = (1 - \rho) \gamma \sum_k \bar{a}_{k,t-1}. \]  \hspace{1cm} (22)

Dividing through by \( \gamma \sum_k \bar{a}_{k,t-1} \), this system of equations can be expressed in terms
of wealth shares:

\[ Q_{1t}^* + \sigma Q_{Pt}^* = \rho \quad \text{and} \]

\[ Q_{Rt}^* + (1 - \sigma)Q_{Pt}^* = (1 - \rho). \]

The system of equations in (23) can be combined with the ROP constraint to generate a solution. First, let \( z_{1t} \) and \( z_{2t} \) satisfy

\[ Q_{P2t}^* = z_{2t}\beta Q_{2P1t-1}^*, \quad \text{and} \]

\[ Q_{1t}^* = z_{1t}\beta Q_{11t-1}^*. \]

The pair \( z_{1t} \) and \( z_{2t} \) are the imputed growth rates relative to the default shares. They determine the scaling up (\( > 1 \)) or down (\( < 1 \)) of claims relative to their default positions.

Under log payoffs for \( P \), the ROP constraint reduces to

\[ (1 - \rho) \log(z_{2t}) + (\rho - \sigma) \log(z_{1t}) = 0. \]

Hence, \( z_{2t} = z_{1t}^{(\sigma - \rho)/(1 - \rho)} \). Combining the ROP constraint with the share equations (23) yields a non-linear equation

\[ z_{1t}^{(\sigma - \rho)/(1 - \rho)} \sigma Q_{P1t-1}^* + z_{1t}Q_{11t-1}^* = \rho/\beta \]

If the common asset is a public asset (\( \sigma < \rho \)) then a solution to (26) exists for strictly positive initial shares. If \( \sigma \) is small enough it may not be unique.\(^{12}\) If there are multiple solutions, the equilibrium will be the largest one. To see why, notice that the public asset valued by both groups and so \( R \) should choose the largest growth rate \( z_1 \) on the public asset consistent with (26). If the common asset is a NIMBY (\( \sigma > \rho \)), then a solution to (26) always exists and is unique.\(^ {13}\)

The solution to (26), coupled with the equation system (24), describes a dynamic equilibrium path for wealth shares with the following qualitative features:

\(^{12}\)Clearly, if \( \sigma = 0 \), then \( z_{1t} = \rho/\beta Q_{11t-1}^* \) is the unique solution. Fixing the value of \( \rho \), there is a neighborhood of \( \sigma = 0 \) such that for each \( \sigma \) in the neighborhood, there are two solutions with the largest approximating \( \rho/\beta Q_{11t-1}^* \) from below as \( \sigma \to 0 \).

\(^{13}\)Apply the Intermediate Value Theorem to the left-hand side of (26) which is strictly increasing in \( z_1 \) from zero when \( \sigma > \rho \).
Proposition 2  Given an equilibrium in the parametric model with ROP durability $\beta < 1$, for all $t > 1$,

(i) The in-group $R$’s share $Q^*_R$ of total wealth is always increasing over time.

(ii) If $\sigma < 1$, then the common asset may be either public or NIMBY, and the in-group’s share of private wealth $S^*_R$ is increasing (and so $S^*_P$ is decreasing). The total share $Q^*_1t$ given to the common asset is weakly increasing over time.

(iii) If $\sigma > 1$ then the common asset is necessarily a NIMBY and the in-group’s share $S^*_R$ of private wealth is decreasing. The total share $Q^*_1t$ given to the NIMBY is weakly decreasing over time.

(iv) If $\sigma = 1$ then the all wealth shares are constant over time.

When combined with the previous proposition, the Proposition implies that the claims $a^*_1t$ and $a^*_2t$ are above their default value and therefore increasing over time. If the common asset is a public asset, then the claim $a^*_P$ lies below its default, implying the out-group’s share of total and private wealth falls over time. However, when the common asset is a NIMBY, then its claim $a^*_P$ is above its default level and so also increasing over time. In that case, its wealth share can rise or fall, depending on the cost $\sigma$ of the NIMBY.

When the common asset is a public asset ($\rho > \sigma$) then the ROP constraint involves a positive trade off for out-group $P$ between the two claims. That is, $z^*_1t > 1$ iff $z^*_2t < 1$. But the influence of in-group $R$ has them choose $z^*_1t > 1$ and $z^*_2t < 1$. This is proved in the Appendix. The wealth share equations (23) show that shares of $P$ decrease while those of $R$ and the public asset increase over time. The rate of appropriation (in shares) depends on $\rho$ and $\sigma$. The higher is $\rho$ the greater the benefit from private appropriation for $R$. The lower is $\sigma$, the higher is the value of the public asset to $P$ and so the easier it is for $R$ to leverage the public asset to enact a private taking.

When the common asset is a NIMBY asset ($\rho < \sigma$), then the ROP constraint involves a negative trade off for $P$ between the two claims. That is, $z^*_1t > 1$ iff $z^*_2t > 1$ and the proof shows in fact that $z^*_1t > 1$ and $z^*_2t > 1$ will hold. So for group $R$ to impose the NIMBY upon group $P$ it must allow some private wealth growth to out-group $P$. Precisely how much it allows depends on parameters. If $\sigma < 1$ then harm of the NIMBY to the out-group is moderate - its cost to $P$ is more modest and so $R$ can increase the NIMBY without acceding too much of the growth in its own private wealth. In that case, both the NIMBY share and group $R$’s private wealth share increase. In other words, both increasing wealth inequality and growth in the NIMBY occur as consistent with Dallas study in Example 3.

If $\sigma > 1$ then the NIMBY then the harm to $P$ is large. This, in turn, limits the leverage $R$ can exert to appropriate private wealth. In that case private wealth inequality
declines even as \( R \)'s share of total wealth increases. By definition, \( R \)'s increase in total share is taken from the potential growth in the NIMBY. We later argue below that this case is atypical.

The parametric model illustrates a key point: the leverage that the dominant group exerts is not primarily determined by whether the common asset is a public asset or a NIMBY. The power to expropriate out-group wealth is similar for values of \( \sigma \) just below and just above \( \rho \). The switching point is, instead, at \( \sigma = 1 \). In other words, \( R \)'s leverage vanishes not when the asset goes from being a public good to being a NIMBY (at \( \sigma = \rho \)) but rather when the negative weight \( \sigma - \rho \) to \( P \) from the NIMBY outweighs positive weight \( 1 - \rho \) to both groups from private wealth. Hence, in the range \((\rho, 1)\), public assets and NIMBY assets have similar qualitative effects on wealth distribution.

### 4.4 Numerical Examples

A few representative graphs illustrate the wealth dynamics of all the cases of Proposition 2. Unless otherwise noted, the figures all display the time paths when the value \( \rho \) to the public asset is set at \( \rho = .5 \), and the durability of right of possession \( \beta \) is set to \( \beta = .953 \) (which is chosen to fully discount a 5% growth rate). The initial shares at \( t = 0 \) are \( Q^*_P = .3 \) and \( Q^*_1 = .5 \).

Figure 1 displays two sets of time paths of \( Q^*_P \) and \( Q^*_1 \), respectively, corresponding to the Out-group’s wealth share and the common asset’s share when \( \rho > \sigma \): the common asset is a public asset. Figure 1a displays the paths when \( \sigma = 0 \). In this case the payoff weights on private and public assets are identical across the groups. The share equations (23) can be combined with the ROP constraint to compute a closed form equilibrium in which \( a^*_{P_{2t}} = \gamma \beta^{1/(1-\rho)} a^*_{P_{2t-1}} \) and so \( Q^*_P = \beta^{1/(1-\rho)} Q^*_{P_{t-1}} \). Notice that the rate of wealth shrinkage \( \beta^{1/(1-\rho)} \) for the out-group is well below the rate \( \beta \) that would occur if the out-group contested a new ownership assignment each period.

Again, the dominant group \( R \) leverages the public asset to extract a share of the growth in private claims from \( P \). As Figure 1a shows, the out-group’s share declines to zero while the public asset share is constant at its steady state value of .5, reflecting its weight in \( R \)'s payoffs.

Alongside Figure 1a is Figure 1b. This figure displays the paths when \( \sigma = .25 \). The common asset is still a public asset, however, this time the out-group places lower value than \( R \) on the public asset. A positive value of \( \sigma \) represents a cost of consuming the returns from the public asset. After an initial adjustment, the out-group’s share in Figure 1b declines less rapidly than in Figure 1a, reflecting \( R \)'s reduced leverage in using the public asset. The ROP constraint is more costly to satisfy. After the adjustment, the public share also increases, returning slowly to its steady state value. In both cases, \( P \)'s private wealth shares declines relative to that of \( R \), as established in Proposition 2.
Figure 1: Time path of Wealth Shares when the Common Asset is a Public Asset. (a) No Costs to Out-group public consumption ($\sigma = 0$). (b) Positive Costs from Out-group public consumption ($\sigma = .25$).

Figure 2 displays the time paths when the common asset is a NIMBY ($\rho < \sigma$). The harm to $P$ measured by $\sigma$ outweighs the benefits measured by $\rho$. Figure 2a displays the shares corresponding to $\sigma = .75$ and $\rho = .5$. These parameters fall under Part (ii) of Proposition 2, namely, the harm inflicted by the NIMBY on out-group $P$ is moderate. In-group $R$ has some leverage to appropriate from $P$. The wealth shares display an initial adjustment followed by a slow decrease in $P$’s private wealth share. Though not displayed, the in-group’s share increases, as per Part (i) in the Proposition. In fact, the dominant group is so successful in appropriating the $P$’s private wealth that it manages to increase both its own share and that of the NIMBY.

Figure 2b displays the shares corresponding to $\sigma = 1.25$. These parameters fall under Part (iii) of Proposition 2. The harm inflicted by the NIMBY on out-group $P$ is high. The cost of the NIMBY to $P$ is relatively higher than the gain to $R$, and so the dominant group is unwilling to sacrifice too much of its own private wealth to increase the NIMBY asset. Thus, while both $R$’s private wealth and the NIMBY increase, the NIMBY share falls, while both $R$’s and $P$’s private wealth share increases. Moreover, the high cost $\sigma$ associated with the NIMBY reduces $R$’s leverage over $P$. The in-group $R$ still manages to increase the level and benefit from the NIMBY but can only do so by allocating a larger portion of the private asset growth to the out-group. As the figure shows, the share to the out-group increases and the share allocated to the NIMBY decreases. The in-group’s share $Q_{rt}$ of total wealth increases as well.

Of the two cases, Figure 2a is the more canonical one. There, $\rho \leq 1/2$ so that private wealth is more prized than public/NIMBY for the in-group. Since $\sigma < 2\rho$ to ensure concavity in the optimization problem, it follows that $\sigma \leq 1$. Applying the Proposition, in the natural case where $\sigma < 1$ the share $S_{P_t}$ of private wealth going to $P$
will decrease and the share going to the NIMBY increases.\footnote{Though the increase in the NIMBY is less than its natural growth rate of $\gamma$.} To summarize:

**Corollary.** If the in-group values its private wealth more than the public/NIMBY asset (i.e., if $\rho < 1/2$), then the out-group’s share of private wealth $Q_{P_t}$ decreases over time, as does its share $Q_{P_t}^*$ of total wealth. The share $Q_{1t}^*$ going to the public/NIMBY asset is weakly increasing over time.

The results in this canonical case are consistent with the Poison-by-zip code example of the Texas Commission on Environmental Quality in Dallas. Locally, both inequality and NIMBY expenditures increased.

\section{Prioritized Right of Possession}

In this Section, we evaluate some consequences of asset prioritization. Formally, asset $k = 1, 2$ will be said to have a priority claim over asset $k'$ in the ROP rule if $\beta_k > \beta_{k'}$. That is, one asset has a priority claim over another in the ROP if it has a more durable right of possession. We maintain most features of the canonical model, but assume that one asset has a priority claim over another.

Consider, an ownership assignment $a$ inherited from the previous period. Let $\beta_1$ denote the durability of possession in the public asset and $\beta_2$ durability in the private asset. Formally, the ROP rule $\pi^\circ$ satisfies $\pi_1^\circ(a) = \beta_1 \gamma a_1$ and $\pi_{2j}^\circ(a) = \beta_2 \gamma a_{j2}$ for $j = R, P$. The assumption $\beta_1 < \beta_2$ is interpreted as courts giving priority to private
property or higher compensation in an attempted public taking. Whereas, $\beta_1 > \beta_2$ indicates courts prioritize the expansion of the public asset in the taking. In fact, the lower is $\beta_2$, the lower the private compensation after the taking. Courts’ designation of a property as “blighted” indicates a low ROP $\beta_2$.\footnote{A ‘Blighted property’ is the legal term for land that is in a dilapidated, unsafe, and unsightly condition. Each state uses different criteria to determine whether property should be classified as blighted.” Legal.match.com (accessed January 24, 2021).}

In general the effects of prioritized claims will depend in subtle ways on the model parameters, including the weights given to the two assets by each of the two groups. Consider again the parametric model in Section 4.3 but with priority claims. Payoffs are again of the form (20). The two assets with identical growth rates, but with one has priority claim over the other ($\beta_1 \neq \beta_2$). It turns out that in the log payoff model, the Dynamic Samuelson condition reduces to the static condition even despite the differential $\beta$s.\footnote{Returning to the Dynamic Samuelson condition, (17), one can verify that with log payoffs, $(\gamma \beta_k) \frac{\partial u_P^{*}(a_t)}{\partial a_k} = \frac{\partial u_P(a_t)}{\partial a_k}$.}

Consequently, the Samuelson condition (21) holds here as well. Defining as before $z_{1t}$ and $z_{2t}$ to be equilibrium, growth rates relative to the default rules, we obtain a straightforward generalization of the equilibrium solution equation (26). Namely,

$$z_1^{(\sigma-\rho)/(1-\rho)} \beta_2 Q_{Pt-1}^* + z_1 \beta_1 Q_{1t-1}^* = \rho$$

(27)

### 5.1 Transfers of Priority Rights

To understand the role of prioritization in this model, a natural thought experiment is analyzed such that, starting from identical priority rights, those rights are transferred to one asset such that the overall right of possession remains unchanged. Specifically, starting from $\beta$ a transfer $\Delta$ is considered such that the right of possession for the assets are now

$$\beta_2 = \beta + \Delta \quad \text{and} \quad \beta_1 = \beta - \Delta$$

(28)

when $\Delta > 0$, the private asset has priority. When $\Delta < 0$, the common asset has priority. Notice that this “neutral transfer” is not actually neutral in the ROP constraint since the out-group may weight one asset more highly than another.

**Proposition 3** In the parametric model, let $\beta$ be the right of possession when neither asset has priority. Consider a a prioritized ROP rule satisfying (28). Then (i) all the results of Proposition 2 continue to hold. (ii) If the common asset is a NIMBY, then at each date $t$, the out-group’s wealth share $Q_{Pt}^*$ under prioritized ROP is larger (smaller) than the non-prioritized ROP if $\Delta > 0$ ($< 0$).

The effects of prioritization change quantitative but not qualitative features of the model. The quantitative changes are most clearcut when the common asset is a NIMBY.
In words, Part (ii) asserts that private asset prioritization helps the out-group, while NIMBY asset prioritization harms it. This follows from the fact that there is no free disposal. A more durable right of possession right to a NIMBY asset is harmful to a group harmed by the NIMBY. Durability of the private asset, on the other hand, always helps the out-group since it improves the default outcome when the assignment is contested. Hence, prioritization of private assets neutralizes in-group power. Prioritization of NIMBY assets enhances in-group power.

The results are quite different when the common asset is a public asset. However, the net effects of prioritization will be subtle. Due to concavity of payoffs the up-weighting one asset will not be offset by an equal down-weight of the other. To see this more clearly, consider the simple case where $\sigma = 0$. Under these parameters, the common asset is public, and the payoffs are identical across the groups. It is straightforward to show that the public asset’s share remains constant over time and the wealth share of the out-group declines according to

$$Q_{P_t} = (\beta - \Delta)\rho/(1-\rho)(\beta + \Delta)Q_{P_{t-1}}$$

(29)

One can verify the following. At every given date, the wealth share of the out-group is lower if (i) the private asset has higher priority ($\Delta > 0$) and has higher weight in the group’s payoffs ($\rho \leq .5$), or (ii) the public asset has higher priority ($\Delta < 0$) and has higher weight in the group’s payoffs ($\rho \geq .5$). Hence, in these cases, the out-group’s wealth share will decline more rapidly under prioritized ROP. In addition, these are sufficient but not necessary conditions. For instance wealth share of the out-group is lower if $\Delta > 0$ even for some $\rho > .5$ provided that $\rho$ is not too close to one. Figure

This suggests that prioritization hurts the out-group for a more robust set of parameters than those that benefit the out-group. Consider, for instance, the fully symmetric case where $\rho = .5$. Private and public assets are equally weighted in both groups’ payoffs. Equation (29) then simplifies to

$$Q_{P_t} = (\beta^2 - \Delta^2)Q_{P_{t-1}}$$

This means that the path when the private asset is prioritized is the same as that when the public asset is prioritized. It is also clear that prioritization - in either direction - benefits the in-group and gives a lower wealth share to the out-group in equilibrium. Because the loss in priority in one asset is out-weighs the gain in the other, prioritization introduces slack in the ROP constraint. Thus prioritization itself has consequences even when the assets are equally valued by both groups.

5.2 Numerical Examples of Prioritization

Asymmetric effects of prioritization arise when $\rho \neq .5$. Figures 3, and 4 illustrate the asymmetries. In each of these figures, the initial out-group wealth share is $Q_{P0} = .3$. 24
The baseline (no-priority) right of possession is $\beta = .8$. The gain/loss in priority is $|\Delta| = .17$.

Figure 3 displays the effects when $\rho = .33$, the private asset is more highly valued. The Figure displays three graphs, one with no prioritization, one where the public asset has higher priority, one where the private asset has higher priority. When the private asset is more valued, the out-group fares worst when the public asset has higher priority. The paring is mutually reinforcing: lower priority for the private asset gives the in-group more leverage in extracting private wealth; higher payoff weight for this asset induces them to use it.

Analogously, Figure 4 displays the effects when $\rho = .67$, the public asset is more highly valued. In this case, the out-group fares worst when the private asset has higher priority. The in-group’s leverage is larger for on the public asset; higher payoff weight for this asset induces them to use it. In this case, however, leverage over the public asset helps the in-group appropriate private wealth.

![Figure 3](image3.png)

**Figure 3:** Time paths of out-group wealth shares when the private asset is more highly valued ($\rho = .33$). Highest wealth appropriation occurs when public asset has priority.
Figure 4: Time paths of out-group wealth shares public asset is more highly valued ($\rho = .67$). Highest wealth appropriation when private asset has priority.

In each case, the in-group benefits from a legal system that prioritizes the least preferred asset. We note the fact that the paths in Figure 4 fall faster than those in Figure 3. Higher payoff weight on the public asset makes it easier for it to be used as leverage to extract private wealth.

6 Literature on Power, Rights, and Appropriation

This paper introduces a theory of local property dynamics that explores the tension between in-group power and out-group legal protections. Legal equality is represented by ROP rules that treat all individuals equally. Political power is asymmetrically allocated a privileged group.

There are a number of theories of redistribution at the national level. Benabou (2000), Bartels (2008), Campante (2010), Gilens (2012), Acemoglu et al. (2015) and many others have described the connections between political power and policies that facilitate wealth capture and redistribution.

Our analysis takes place at local level. The literature on local public and urban
policies (in Footnote 1) describes the effects of zoning and land use laws. Minimum density requirements limit access to public amenities. NIMBY zoning places landfills near poorer neighborhoods. Takings disproportionately affect marginalized groups. These policies all have implications for property and wealth accumulation.

Almost all the literature is empirical. There is less formal modeling to explicitly connect local political motives and power to wealth extraction. Our theory centers on the local exercise of political power under ostensibly neutral legal constraints. On paper these constraints allow, for instance, property holders to contest a proposed taking and/or the proposed compensation. In practice, however, these constraints may not be much of a check on local power. The legal literature examines rules governing standing (connection to the harm) and on compensation for affected parties when property rights are contested. The literature also analyzes how certain interpretations of takings law may result in under-compensation of property holders (Blume and Rubinfeld (1984), Merrill (1986), Barton H. Thompson (1990), Fennell (2005), Heller and Hills (2008), Kelly (2009), Libecap (2011), and Lee (2013)).

Under-compensation in the model corresponds to a nondurable ROP, i.e., \( \beta_k < 1 \). Heller and Hills (2008), in particular, offer a normative analysis (using our terminology) of \( \beta_k < 1 \), asking “whether landowners ought to receive any share of the increased value resulting from the land assembly itself” (p.1477). The extreme case of a status quo right of possession, an individual who is, say, excluded from a neighborhood due to restricted zoning would be excluded from all subsequent appreciation in housing values occurred in the excluded area.

The right of possession, as modeled here is related to the status quo effects in models of legislative bargaining. The distinction between durable and status quo ROPs here can compared to the distinction in Bowen et al. (2014) between mandatory and discretionary status quo programs in legislatures. In their model a discretionary program is one where funding fully expires by the next legislative session, whereas a mandatory program is one where current funding is the baseline upon which future decisions are made. These are both default rules - rules that go into effect when an agreement between factions, parties, etc., breaks down. A status quo ROP is analogous to a discretionary rule in the sense that they both constitute the weakest constraint on current power. The durable ROP is analogous to a mandatory rule in the sense that they both constitute the strongest constraint on current power. In our study the rules apply to propertied assets rather than funding flows, and this allows us to directly analyze property and wealth dynamics.

In practice a “right of possession” is a legal principle rather than a formal code, typically encompassing a diverse collection of precedents and statutes. Libecap (2011) and Leonard and Libecap (2015) describe the principle (“first-possession”) in use for

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17While a “taking” normally refers to the re-assignment of private for public purposes (eminent domain laws), takings can and have applied used to put in place private-to-private transfers as well (Supreme Court decisions *Berman V. Parker* and *Kelo v. City of New London*)

18See Eraslan et al. (2020) for a survey.
fishing rights. Generally, the ROP, is like any default outcome, is just the off-path node in the absence of agreement. The result for out-groups can be severe if the default rule is coercive. Acemoglu and Wolitsky (2011) introduce a model of labor coercion in which coercion arises if employers use force or threat of force to make agents accept contracts that they would not otherwise accept. Cao and Lagunoff (2020) examine wealth expropriation in autocracies where the “default result” is whatever happens in the absence of commitment to property enforcement. In these models, the implied default outcome is provisional and state dependent. In our prior paper, the lack of enforcement of and commitment to the rule of law in an autocracy results in a tragedy of the commons problem that would not arise here.

Even as a general principle, a ROP is a commitment to the rule of law. While the ROP provides greater protection to out-groups, the pattern of ownership will still be heavily influenced by asymmetries in local political power, and by a lack of long run commitment by ruling parties to existing ownership claims. In this sense, the failure of legal norms to check the asymmetries in political power can be compared to other forms of institutional bias in, for instance, Benabou (2000), Bartels (2008), and Campante (2010).

The model also highlights the role of prioritization of assets. Prioritization is an old topic in law and finance. Less is known about the distributional effects of prioritization in property appropriation. This is due in part because ROP rules are highly non-uniform across states and nations. Fennell (2005) and Heller and Hills (2008) allude to inefficiencies and inequities arising from the ways different stake holders are prioritized in eminent domain cases. Pistor (2019) gives a compelling account of how prioritization in the “coding of capital” has historically transformed ownership and wealth distribution. She describes how priority rules on bankruptcy, incorporation, municipal designations, and credit have benefitted powerful groups at the expense of others. MacFarlane (1978) famously studied seventeenth century land enclosures that were aided by laws and norms that prioritized farming over grazing rights.

Finally, the paper’s emphasis on the aggressive power of dominant groups to alter property rights contrasts somewhat with “Coasian” view in which politico-economic forces favor inertia and impede efficiency-improving changes. Acemoglu (2003), Libecap (2011), and Vahabi (2011), among others cite a number of reasons for the failure, including inegalitarian effects of monopolized political power, and the inability to commit in the long run. “Underlying the Coase theorem is the ability to write enforceable contracts... widespread enforcement problems arise because most contracts are enforced by the state. Contracts that the state, or social groups controlling the state, would like to write with others, e.g., the citizens, will be non-enforceable by definition because groups controlling the state cannot commit to not using their power to renege on their promises or to not changing the terms of the contract.” (Acemoglu (2003), p.622). Failure of

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19 See for instance Rollison (1932).
policy due to lack of commitment by political rulers and policy makers is a ubiquitous theme dating back at least to Kydland and Prescott seminal work. Yet, dynamic consistencies are also a function of political monopoly. Competing power allows for strategic partnerships. Standard Folk Theorem arguments demonstrate how this is done.

In both the “raw power dynamics” and “Coasian” views of property rights, politically dominant groups work the system and produce inefficient property rights. In the Coasian view, these groups delay or prevent an efficient assignment. In the current model, aggressive change results in an inefficient consumption path. The underlying reasons for both sources of inefficiency are similar. In the Coasian world, the inability of the political system to commit to compensate groups harmed by change (transactions costs) results in delay. In the present model, inability of in-group R to commit to an assignment, once and for all, results in inefficient consumption smoothing. The in-group R would be better off while still satisfying the ROP constraint if it could enact a large property redistribution in the beginning, then stick to it from that point onward.

One might ask then why legal system do not adopt durable rights of possession or, alternatively, why the polities cannot find ways to fully commit to an ownership assignment. In our view, commitment is a result of joint enforcement of norms. The monopolization of political power prevents this. Monopolized power is more common at the local than the national level. Property rights are often set at the local level. Many of the most commonly used mechanisms - land use regulation and zoning - are determined locally. However, recourse against local power is determined by the legal system, a separate, often supra-local, branch of government.

As in any work, the model omits some relevant factors. It does not include endogenous growth or market responses to political decisions (as in empirical work of Glaeser and Gyourko (2002)). Nor does it consider rotation of power. Further development is needed to analyze effects of lobbying, political contributions, corruption, and explicitly discriminatory norms such as racial red-lining and non-enforcement of civil rights. These factors would appear to reinforce the bottom line. Ultimately, there is much more to unpack before the full effects of power and privilege are understood.

References


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7 Appendix

7.1 Proof of Theorem 1, the Generalized Samuelson condition

Using Euler equation (14) for each of two ownership claims $jk$ and $j'k'$, we obtain

$$
\frac{\partial u_R}{\partial a_{j'k't}} + \varphi_t \frac{\partial u_P}{\partial a_{j'k't}} = \frac{\mu_t - \gamma}{\mu_t - \delta} \left[ (\varphi_t - \varphi_{t+1}) \frac{\partial V_P}{\partial a_{j'k't}} + \gamma k' \mu_{t+1} \right]
$$

(30)

Observe that because the ROP default rule $\pi^o$ is non-discriminatory, $\beta_{R2} = \beta_{P2} = \beta_2$ and so it follows from the resource constraint that $\frac{\partial V_P}{\partial a_{P2t}} = -\frac{\partial V_P}{\partial a_{R2t}}$. In terms of the public/NIMBY asset $k = 1$ and private asset $k = 2$ we obtain from (30):

$$
\frac{\partial u_R}{\partial a_{j1t}} + \frac{\partial u_R}{\partial a_{R2t}} + \frac{\partial u_P}{\partial a_{j1t}} + \frac{\partial u_P}{\partial a_{P2t}} = \frac{\mu_t - \gamma}{\mu_t - \delta} \left[ (\varphi_t - \varphi_{t+1}) \frac{\partial V_P}{\partial a_{j1t}} + \gamma 1 \mu_{t+1} \right]
$$

(31)

where $j = R, P$.

Now apply the Euler equations in (14) for our public/NIMBY asset $k = 1$ and private asset $k = 2$. Observe that we can eliminate the multipliers $\mu_t$ and $\mu_{t+1}$ on the resource constraints by substituting $a_{P2t} = \sum_k \gamma \bar{a}_{kt} - \sum_{k \neq 2} \bar{a}_{kt} - a_{R2t}$ directly into the payoff $u_P$ for group $P$ (note that $a_{P2t}$ does not enter the payoff of group $R$). The relevant Euler equations in $k = 1$ and $k = 2$ become

$$
\frac{\partial u_R}{\partial a_{1t}} = - \left( \varphi_t \frac{\partial u_P}{\partial a_{1t}} \right) + \delta (\varphi_t - \varphi_{t+1}) \frac{\partial V_P}{\partial a_{1t}}
$$

$$
+ \varphi_t \frac{\partial u_P}{\partial a_{P2t}} + \delta (\varphi_t - \varphi_{t+1}) \frac{\partial V_P}{\partial a_{P2t}}
$$

(32)

$$
\frac{\partial u_R}{\partial a_{R2t}} = \varphi_t \frac{\partial u_P}{\partial a_{P2t}} + \delta (\varphi_t - \varphi_{t+1}) \frac{\partial V_P}{\partial a_{P2t}}
$$

Substituting and rearranging terms, we obtain

$$
\frac{\partial u_R}{\partial a_{j1t}} + \frac{\partial u_R}{\partial a_{R2t}} + \frac{\partial u_P}{\partial a_{1t}} + \delta (1 - \varphi_{t+1}/\varphi_t) \frac{\partial V_P}{\partial a_{1t}} = 1
$$

(33)

Finally substitute the expression for $\frac{\partial V_P}{\partial a_{P2t}}$ in equation (15) to arrive at the general, Dynamic Samuelson condition.

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20This relies on the fact that asset 2 is exclusively private.
As for the second part of the Theorem, let \( \beta = \beta_1 = \beta_2 \). By (7),

\[
\frac{\partial V^\circ_p(a_t)}{\partial a_{jkt}} = \sum_{\tau=0}^{\infty} \delta^\tau \frac{\partial u^{\circ \tau}_p(a_t)}{\partial a_{jkt}} = \frac{\partial g_p(a_t)}{\partial a_{jkt}} \sum_{\tau=0}^{\infty} \delta^\tau f((\beta \gamma)^\tau) \tag{34}
\]

where \( f \) and \( g \) are the homotheticity scale functions in \( u_j(\epsilon a) = f_j(\epsilon) g_j(a) \) assumed at the outset in the model description. Substituting (34) into (33) and using the homotheticity assumption above we obtain

\[
\frac{\partial g_R/\partial a_{j1t}}{\partial g_R/\partial a_{R2t}} + \frac{\partial g_p/\partial a_{1t}}{\partial g_p/\partial a_{P2t}} (1 + \delta(1 - \varphi_{t+1}/\varphi_t) \sum_{\tau=0}^{\infty} \delta^\tau f((\beta \gamma)^\tau)) = 1
\]

\[
\Rightarrow \frac{\partial g_R/\partial a_{j1t}}{\partial g_R/\partial a_{R2t}} + \frac{\partial g_p/\partial a_{1t}}{\partial g_p/\partial a_{P2t}} = 1 \tag{35}
\]

which is equivalent, by the homotheticity assumption, to

\[
\frac{\partial u_R/\partial a_{j1t}}{\partial u_R/\partial a_{R2t}} + \frac{\partial u_p/\partial a_{1t}}{\partial u_p/\partial a_{P2t}} = 1 \tag{36}
\]

This establishes the static Samuelson condition as a special case of the dynamic Samuelson condition when \( \beta_k = \beta \).

\[
\]

7.2 Preliminaries for the Propositions in the Canonical Model

Let \( \eta_t = (\eta_{1t}, \eta_{R2t}, \eta_{P2t}) \). For the following Lemma, let \( \eta_k = \pi^*_k(a_{t-1}) - \pi^*_k(a_{t-1}) = a^*_k - \beta \gamma a_{t-1} \), the difference between the equilibrium and default claims. Denote \( MRS^*_j(a_t) = \frac{\partial u_j(a_t)/\partial a_{1t}}{\partial u_j(a_t)/\partial a_{2t}} \) for each group \( j \).

Lemma 1 (Equilibrium relative to a default rule) Given the canonical model, let \( \pi^* \) be an equilibrium assignment with equilibrium path \( \{a^*_t\} \). Let \( \pi^0 \) be a default rule with right of possession \( \beta < 1 \). Then

1. Suppose \( k = 1 \) is a public asset. Then \( MRS_R(a^*_t) > MRS_R(a^*_t) \) (group R’s MRS increases over time), and \( \eta_{P2} > 0 \) implies \( \eta_{R2} > 0 \), and \( \eta_1 < 0 \) implies \( \eta_{P2} > 0 \). That is, relative to the default, an increase private wealth in equilibrium by the out-group implies that private wealth increases for the in-group as well (though not the converse). If public wealth declines, then the private wealth of both groups must increase.

2. Suppose asset \( k = 1 \) is a NIMBY asset for group \( P \) (\( \partial u_P/\partial a_1 < 0 \)). Then \( MRS_R(a^*_t) > MRS_R(a^*_t) \) iff \( MRS_P(a^*_t) < MRS_P(a^*_t) \), and either \( \eta_1 < 0 \) and \( \eta_{P2} < 0 \) or \( \eta_1 > 0 \) and \( \eta_{P2} > 0 \). That is, both the NIMBY asset and private wealth of the out-group \( P \) either both lie above, or both lie below the default values.
3. In either the public or NIMBY asset model, \( a_{R2t} > \beta \gamma a^*_{R2t-1} \) (\( R \)'s claim lies above its default value).

**Proof.** We first prove Part (1). We first show that in any assignment \( a_t \) for which there is slack in both takings and resource constraints, it follows that \( \text{MRS}_R(a_t) + \text{MRS}_P(a_t) < 1 \). Suppose otherwise: suppose there is slack in both constraints under \( \hat{a}_t \). Then \( R \) can improve its payoff by increasing \( a_{R2t} \) which increases \( \text{MRS}_R \) without reducing \( \text{MRS}_P \). This is a contradiction since it increases \( \text{MRS}_R(a_t) + \text{MRS}_P(a_t) \) which is already supposed greater than one.

Next, consider \( \hat{a}_{1t} \) as a public asset. Let \( \hat{a}_t \) be an assignment that satisfies \( \hat{a}_t = \gamma \hat{a}_t^* \), \( \hat{a}_{R2t} = \gamma a^*_{R2t-1} \), and \( \hat{a}_{P2t} = \gamma a^*_{P2t-1} \). Clearly, this assignment \( \hat{a}_t \) satisfies both the resource and ROP constraint with slack. Moreover, by homothetic preferences, \( \text{MRS}_R(\hat{a}_t) = \text{MRS}_R(a^*_t) \), and \( \text{MRS}_P(\hat{a}_t) < \text{MRS}_R(a^*_t) \). Since \( a^*_t \) satisfies the Samuelson condition in \( t - 1 \), clearly \( \text{MRS}_R(\hat{a}_t) + \text{MRS}_P(\hat{a}_t) < 1 \). Because of the slack in both constraints, group \( R \) increases its payoff at \( t \) by increasing \( a_{R2t} \) above \( \hat{a}_{R2t} = \gamma a^*_{R2t-1} \), well above its default value. Hence, \( \text{MRS}_R(a_t) > \text{MRS}_R(\hat{a}_t) = \text{MRS}_R(a^*_t) \). At this new \( a_t \), \( \text{MRS}_R(\hat{a}_t) + \text{MRS}_P(\hat{a}_t) > \text{MRS}_R(a_t) + \text{MRS}_P(\hat{a}_t) \) moving closer to the Samuelson condition, while still satisfying the constraints. Finally, because of homotheticity, \( \hat{a}_{1t} \) cannot be scaled up to the same extent in order to satisfy the Samuelson condition.

Part (2). Next, consider \( \hat{a}_{1t} \) as a NIMBY asset. Now suppose \( \hat{a}_t \) satisfies \( \hat{a}_t = \gamma \hat{a}_t^* \), \( \hat{a}_{R2t} = \gamma a^*_{R2t-1} \), and \( \hat{a}_{P2t} = \gamma a^*_{P2t-1} \). Then, again, by homothetic preferences, \( \text{MRS}_R(\hat{a}_t) = \text{MRS}_R(a^*_t) \), and once again, \( \text{MRS}_P(\hat{a}_t) < \text{MRS}_R(a^*_t) \), in this case it is due to the fact that \( \text{MRS}_P < 0 \) because the \( k = 1 \) asset is a NIMBY. Again, since \( a^*_t \) satisfies the Samuelson condition in \( t - 1 \), clearly \( \text{MRS}_R(\hat{a}_t) + \text{MRS}_P(\hat{a}_t) < 1 \). Again, there is slack in both constraints, and so group \( R \) should increase the value \( a_{R2t} > \hat{a}_{R2t} \). In so doing, it increases \( \text{MRS}_R(a_t) \) and moves the left-hand side of the Samuelson condition closer to one, while still satisfying the constraints. Notice that this implies \( a^*_{R2t} > \beta \gamma a^*_{R2t-1} \), i.e., \( a^*_{R2t} \) lies above its default value.

Now suppose instead that \( \hat{a}_t \) satisfies \( \hat{a}_t = \gamma \hat{a}_t^* \), \( \hat{a}_{R2t} = \gamma a^*_{R2t-1} \), and \( \hat{a}_{P2t} = \gamma \beta a^*_{P2t-1} \). Again, we have \( \text{MRS}_R(\hat{a}_t) = \text{MRS}_R(a^*_t) \), and in this case \( \text{MRS}_P(\hat{a}_t) > \text{MRS}_R(a^*_t) \). This assignment thus violates both the ROP constraint and the Samuelson condition. To satisfy the constraint and the Samuelson condition \( \hat{a}_{1t} \) must be decreased and/or \( \hat{a}_{P2t} \) must be increased. By homotheticity, in order to satisfy the Samuelson condition \( a_{P2t} \) must be increased relatively more than the decrease in \( \hat{a}_{1t} \), and since there is slack in the resource constraint under \( \hat{a}_t \) this can be done.

For each of these initial claims, the co-movements of the MRS of each group are negatively related.

Part (3). This statement follows from the proofs of Parts (i) and (ii).

### 7.3 Proofs of Propositions in the Canonical Model

**Proof of Proposition 1.**

We prove Part (i) first. Simplify notation so that \( a^*_{jk} = \beta \gamma a^*_{jk,t-1} \) for \( j = R, P \) and \( k = 1, 2 \).
Notice that by homotheticity,

\[
\frac{\partial u_R(a^*_t)/\partial \bar{a}_1}{\partial u_R(a^*_t)/\partial a_{R2}} + \frac{\partial u_P(a^*_t)/\partial \bar{a}_1}{\partial u_P(a^*_t)/\partial a_{P2}} = \frac{\partial u_R(\beta \gamma a^*_{t-1})/\partial \bar{a}_1}{\partial u_R(\beta \gamma a^*_{t-1})/\partial a_{R2}} + \frac{\partial u_P(\beta \gamma a^*_{t-1})/\partial \bar{a}_1}{\partial u_P(\beta \gamma a^*_{t-1})/\partial a_{P2}}
\]

(37)

\[
= \frac{\partial u_R(a^*_{t-1})/\partial \bar{a}_1}{\partial u_R(a^*_{t-1})/\partial a_{R2}} + \frac{\partial u_P(a^*_{t-1})/\partial \bar{a}_1}{\partial u_P(a^*_{t-1})/\partial a_{P2}} = 1.
\]

Suppose first \(a^*_{P2t} > \beta \gamma a^*_{R2t-1} \equiv \bar{a}^*_{P2t}\). By definition, \(\eta_{P2t} > 0\). By Lemma 1, \(\eta_{R2t} > 0\), so \(a^*_{R2t} > \bar{a}^*_{R2t} = \beta \gamma a^*_{R2t-1}\). By Lemma 1 we also have \(\eta_{1t} < 0\) so that \(\bar{a}^*_1 < \bar{a}^*_1 = \beta \gamma a^*_1\). Using (37) we obtain

\[
\frac{\partial u_R(a^*_t)/\partial \bar{a}_1}{\partial u_R(a^*_t)/\partial a_{R2}} + \frac{\partial u_P(a^*_t)/\partial \bar{a}_1}{\partial u_P(a^*_t)/\partial a_{P2}} > \frac{\partial u_R(a^*_t)/\partial \bar{a}_1}{\partial u_R(a^*_t)/\partial a_{R2}} + \frac{\partial u_P(a^*_t)/\partial \bar{a}_1}{\partial u_P(a^*_t)/\partial a_{P2}} = 1
\]

(38)
a contradiction of the Samuelson condition.

Hence assume, without loss of generality, that \(a^*_{P2t} < \beta \gamma a^*_{P2t-1} \equiv \bar{a}^*_{P2t}\). Since \(\eta_{P2t} < 0\) it follows that \(\eta_{1t} > 0\) so that \(\bar{a}^*_1 > \bar{a}^*_1 = \beta \gamma a^*_1\). Thus we have

\[
\frac{Q^*_{Pt}}{Q^*_{1t}} = \frac{a^*_{Pt}}{\bar{a}^*_1} < \frac{\beta \gamma a^*_{Pt-1}}{\beta \gamma a^*_1} = \frac{Q^*_{Pt-1}}{Q^*_{1t-1}}
\]

This also implies that \(MRS_{Pt}(a^*_t) < MRS_P(\beta \gamma a^*_{t-1}) = MRS_{Pt-1}\) (the last equality following from homotheticity). To obtain Samuelson condition we therefore need \(MRS_R(a^*_{t}) > MRS_{Rt}(\beta \gamma a^*_{t-1}) = MRS_{Rt-1}(a^*_t)\). This cannot happen if \(a^*_{R2t} < \beta \gamma a^*_{R2t-1}\). In fact, to satisfy the resource constraint without slack, either \(a^*_{R2t} > \gamma a^*_{R2t-1}\) or \(\bar{a}^*_1 > \gamma a^*_1\) or both. In order for \(MRS_R(a^*_t) > MRS_{Rt-1}(a^*_t)\) we must have \(a^*_{R2t} > \gamma a^*_{R2t-1}\). Straightaway this implies \(\frac{Q^*_{Pt}}{Q^*_{1t}} > \frac{Q^*_{Rt-1}}{Q^*_{1t-1}}\).

It remains to show \(\frac{Q^*_{Pt}}{Q^*_{1t}} > \frac{Q^*_{Rt-1}}{Q^*_{1t-1}}\) or, equivalently, \(\frac{a^*_{R2t}}{\bar{a}^*_1} > \frac{a^*_{R2t-1}}{\bar{a}^*_1}\). Letting \(\phi\) satisfy \(\bar{a}^*_1 = \phi \bar{a}^*_1\), homotheticity and \(MRS_R(a^*_t) > MRS_{Rt-1}(a^*_t)\) implies \(a^*_{R2t} > \phi a^*_{R2t-1}\). Consequently,

\[
\frac{a^*_{R2t}}{\bar{a}^*_1} > \frac{\phi a^*_{R2t-1}}{\phi \bar{a}^*_1}
\]

which concludes the proof of part (1).

For Part (ii) apply the Lemma.

\[\blacksquare\]

**Proof of Proposition 2**.

Flow payoffs are \(u_R = \rho \log(\bar{a}_{1t}) + (1 - \rho) \log a_{R2t}\) for \(R\) and \(u_P = (\rho - \sigma) \log(\bar{a}_{1t}) + (1 - \rho) \log a_{P2t}\) for \(P\). Here, \(\rho > \sigma\) indicates a public good, \(\rho < \sigma\) corresponds to a NIMBY good, and \(\sigma < 1(>1)\) indicates a NIMBY good whose downside for \(P\) is relatively
smaller (larger) than the value of private wealth. Assume $\sigma < 2\rho$ to guarantee concavity of the joint preference across the two groups.

The Samuelson condition is

$$\left( \frac{\rho}{1-\rho} \right) \frac{a_{R2t}^*}{\bar{a}_{1t}^*} + \left( \frac{\rho - \sigma}{1-\rho} \right) \frac{a_{P2t}^*}{\bar{a}_{1t}^*} = 1$$

Combining the resource constraint $\bar{a}_{1t}^* = \gamma \sum_k \bar{a}_{kt-1}^* - a_{R2t}^* - a_{P2t}^*$ with the Samuelson condition (39) we obtain the system of equations

$$\bar{a}_{1t}^* + \sigma a_{2Pt}^* = \rho \gamma \sum_k \bar{a}_{kt-1}^*, \text{ and}$$

$$a_{R2t}^* + (1-\sigma)a_{P2t}^* = (1-\rho)\gamma \sum_k \bar{a}_{kt-1}^*.$$

In terms of total wealth shares of each claim, we obtain

$$Q_{1t}^* + \sigma Q_{Pt}^* = \rho \text{ and}$$

$$Q_{Rt}^* + (1-\sigma)Q_{Pt}^* = (1-\rho).$$

For general values of $\rho$ and $\sigma$, the model does not admit a closed form solution. Drop time subscripts and let $z_1$ and $z_2$ satisfy $a_{P2t}^* = z_2 \gamma \beta a_{2Pt-1}^*$ and $\bar{a}_{1t}^* = z_1 \beta \gamma \bar{a}_{1t-1}^*$. Substituting these into (40) yields

$$z_1 \beta \gamma \bar{a}_{1t-1}^* + \sigma z_2 \gamma \beta a_{2Pt-1}^* = \rho \gamma \sum_k \bar{a}_{kt-1}^*$$

Next, observe that the ROP constraint under the given payoffs reduces to

$$(1-\rho)\log(z_2) + (\rho - \sigma)\log(z_1) = 0.$$

or $z_2 = \frac{z_1^{(\sigma-\rho)/(1-\rho)}}{z_1^{\sigma-\rho}}$. Notice that $\sigma < \rho$ (public asset), then $z_2 > 1$ iff $z_1 < 1$. And if $\sigma > \rho$ (NIMBY asset) then $z_2 > 1$ iff $z_1 > 1$. That is, private wealth is above its default value $\beta \gamma a_{P2t-1}$ whenever a public asset is below default value $\beta \gamma \bar{a}_{1t-1}$ and whenever a NIMBY asset is above its default value $\beta \gamma \bar{a}_{1t-1}$.

Substituting again yields

$$z_1 \beta \bar{a}_{1t-1}^* + \sigma z_1^{(\sigma-\rho)/(1-\rho)} \beta a_{2Pt-1}^* = \rho \sum_k \bar{a}_{kt-1}^*$$

(43)
a nonlinear equation in \( z_1 \) as a function of current states \( \bar{a}_{1t-1}^* \) and \( a_{2pt-1}^* \). Dividing both sides by total wealth \( \sum_k \bar{a}_{kt-1}^* \) we obtain the equation for \( z_1 \) in terms of the wealth shares

\[
z_1^{(\sigma - \rho)/(1 - \rho)} \sigma Q_{pt-1}^* + z_1 Q_{1t-1}^* = \rho/\beta
\]  

(44)

By the implicit function Theorem, a solution \( z_1^* (\sigma, \rho, \beta, Q_{pt-1}^*, Q_{1t-1}^*) \) exists and is increasing in \( \rho \), and decreasing in \( \sigma, \beta, Q_{1t-1}^* \), and \( Q_{pt-1}^* \).

Now let \( \bar{z}_1 = \min \{ z_1, z_1^{(\sigma - \rho)/(1 - \rho)} \} \) and \( \bar{z}_1 = \max \{ z_1, z_1^{(\sigma - \rho)/(1 - \rho)} \} \). Then,

\[
\bar{z}_1 \beta (\bar{a}_{1t-1}^* + \sigma a_{2pt-1}^*) \geq \rho \sum_k \bar{a}_{kt-1}^* \geq \bar{z}_1 \beta (\bar{a}_{1t-1}^* + \sigma a_{2pt-1}^*).
\]

Using the first equation in (40), \( \bar{a}_{1t-1}^* + \sigma a_{2pt-1}^* = \rho \sum_k \bar{a}_{kt-1}^* \) so that

\[
\beta \bar{z}_1 \geq 1 \geq \beta \bar{z}_1
\]  

(45)

and so \( \bar{z}_1 \geq 1/\beta > 1 \). If \( \sigma < \rho \), then (45) and Proposition 1 implies \( z_1 > 1 \) and \( z_2 < 1 \), and consequently, \( z_1 > 1/\beta > z_2 \). If \( \sigma > \rho \), then (45) implies \( z_1 > 1 \) and \( z_2 > 1 \). In this second case, if \( \sigma = 1 \) we obtain \( z_1 = z_2 = 1/\beta \). For \( \sigma > 1 \) we obtain \( z_2 > 1/\beta > z_1 \) and for \( \sigma < 1 \) we obtain \( z_2 < 1/\beta < z_1 \).

Putting these facts together, consider the equation system in (40). We obtain

\[
Q_{1t}^* = \rho - \sigma Q_{pt}^* = \rho - \sigma \beta z_2 Q_{pt-1}^* > \rho - \sigma Q_{pt-1}^* = Q_{1t-1}^* \text{ if } \sigma < 1
\]

\[
Q_{1t}^* = \rho - \sigma Q_{pt}^* = \rho - \sigma \beta z_2 Q_{pt-1}^* < \rho - \sigma Q_{pt-1}^* = Q_{1t-1}^* \text{ if } \sigma > 1
\]

and

\[
Q_{R2t}^* = (1 - \rho) - (1 - \sigma) Q_{pt}^* = (1 - \rho) - (1 - \sigma) \beta z_2 Q_{pt-1}^* > (1 - \rho) - (1 - \sigma) Q_{pt-1}^* = Q_{1t-1}^* \text{ if } \sigma < 1
\]

\[
= Q_{1t-1}^* \text{ if } \sigma < 1
\]

\[
Q_{R2t}^* = (1 - \rho) - (1 - \sigma) Q_{pt}^* = (1 - \rho) - (1 - \sigma) \beta z_2 Q_{pt-1}^* > (1 - \rho) - (1 - \sigma) Q_{pt-1}^* = Q_{1t-1}^* \text{ if } \sigma > 1, \text{ and}
\]

\[
Q_{R2t}^* = Q_{R2t-1}^* \text{ if } \sigma = 1
\]
Note that these cases imply that $Q_{R2t}^*$ and $Q_{1t}^*$ are always increasing over time (and $Q_{P2t}^*$ decreasing) when $\rho > \sigma$, i.e., when $k = 1$ is a public asset. In the case of a NIMBY asset $Q_{R2t}^*$ is always increasing, $Q_{1t}^*$ and $Q_{P2t}^*$ move in opposite directions over time depending on whether $\sigma > 0$ or $< 1$.

As for purely private shares, it is clear that $S_{Rt}^*$ is increasing when $\rho > \sigma$, i.e., when $k = 1$ is a public asset. So suppose without loss of generality $\rho < \sigma$ (NIMBY). Then

$$S_{Rt}^* \equiv \frac{a_{R2t}}{a_{R2t} + a_{P2t}} = \frac{(1 - \rho)(a_{R2t} + a_{P2t} + \bar{a}_{1t}) + (\sigma - 1)a_{P2t}}{a_{R2t} + a_{P2t} + \bar{a}_{1t} - \bar{a}_{1t}}$$

$$= \frac{(1 - \rho)\gamma(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + (\sigma - 1)\bar{z}_2\beta a_{P2t-1}}{\gamma[(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) - \rho(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + \sigma z_2\beta a_{P2t-1}]}$$

$$= \frac{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + (\sigma - 1)\bar{z}_2\beta a_{P2t-1}}{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + \sigma z_2\beta a_{P2t-1}}$$

if $\sigma < 1$, $> \frac{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + (\sigma - 1)a_{P2t-1}}{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + \sigma a_{P2t-1}} = S_{Rt-1}^*$, and

if $\sigma > 1$, $< \frac{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + (\sigma - 1)a_{P2t-1}}{(1 - \rho)(a_{R2t-1} + a_{P2t-1} + \bar{a}_{1t-1}) + \sigma a_{P2t-1}} = S_{Rt-1}^*$

To understand the first inequality, notice that we established that $z_2\beta < 1$ holds whenever $\sigma < 1$. Consequently, treating $z_2\beta$ as a variable, increasing it reduces the value of the right-hand side expression. By contrast we established that $z_2\beta > 1$ holds whenever $\sigma > 1$. In that case, letting $x = z_2\beta$, the last inequality is expressed as

$$\frac{a + bx}{a + cx} < \frac{a + b}{a + c}$$

where $a, b, c > 0$, $b < c$ and $x > 1$. For these parameter restrictions, one can readily verify this inequality. To conclude, we have shown that private wealth share of $R$ increases over time when $\sigma < 1$ and decreases over time when $\sigma > 1$. We conclude the proof.

## 7.4 Proofs of Propositions in the Prioritized ROP Model

### Proof of Proposition 3.

Part (i). Consider the parameterized model with prioritized ROP. With different priority rights, we nevertheless repeat many of the steps in the proof of the earlier Proposition 2. In particular, the same Samuelson condition (21) holds, as do the equations in (22). Again dropping time subscripts, define the scale variables $z_1$ and $z_2$ for the priority model as $a_{P2t}^* = z_2\gamma_0\beta_2 a_{P2t-1}^*$ and $\bar{a}_{1t} = z_1\beta_1\gamma\bar{a}_{1t-1}$. Once again the ROP constraint implies $z_2 = z_1^{(\gamma - \rho)/(1 - \rho)}$.
Substituting these into the first equation in (22) yields Equation (27), repeated for convenience here as

\[ z_1 \beta_1 \gamma Q^*_1 t_{-1} + \sigma z_1^{(\sigma-\rho)/(1-\rho)} \beta_2 Q^*_p t_{-1} = \rho. \]  

(46)

This equation is a generalization of (26). Again using similar arguments as in the proof of Proposition 2, define

\[ \underline{x} = \min \{z_1 \beta_1, z_1^{\rho/(1-\rho)} \beta_2 \} \]  

and

\[ \bar{x} = \max \{z_1 \beta_1, z_1^{\rho/(1-\rho)} \beta_2 \}. \]

Then,

\[ \bar{x} (Q^*_1 t_{-1} + 2 \rho Q^*_p t_{-1}) \geq \rho \geq \underline{x} (Q^*_1 t_{-1} + 2 \rho Q^*_p t_{-1}). \]

Again using the share equations in (22), we obtain

\[ \bar{x} \geq 1 \geq \underline{x} \]

From these and the definitions of \( z_1 \) and \( z_2 \), it follows that \( z_1 > 1 \) and \( z_2 > 1 \) if \( \sigma > \rho \).

And once again, if \( \sigma > 1 \) then \( z_2 \beta_2 > 1 > z_1 \beta_1 \); if \( \sigma < 1 \) then \( z_2 \beta_2 < 1 < z_1 \beta_1 \). If \( \sigma = 1 \) then \( z_2 \beta_2 = z_1 \beta_1 \).

Thus only the net effects \( z_k \beta_k \) can be discerned, but the arguments in Proposition 2 always compared \( z_k \beta \) to one, and so the comparisons with \( Q^*_t \) and \( Q^*_t \) remain the same in the present result.

Observe in this case that \( \beta_1 \) and \( \beta_2 \) can be varied independently, and so using (46), the solution \( z^*_1 \) is decreasing in \( \beta_1 \) and \( \beta_2 \) independently, but is also decreasing in the ratio \( \beta_2 / \beta_1 \).

Part (ii). Since \( z_1 t \) is increasing in \( \beta_1 \) and \( z_2 t = z_1^{(\sigma-\rho)/(1-\rho)} \) increasing in \( \beta_2 \), if \( \Delta > 0 \), then \( Q^*_t \) is higher than if \( \Delta = 0 \). And if \( \Delta < 0 \), then \( Q^*_t \) is higher than if \( \Delta = 0 \). Since asset 2 is a NIMBY, the left-hand side of (46) is increasing. Hence, it cannot be the case that both \( Q^*_p t = z_2 t \beta_2 Q^*_p t_{-1} \) and \( Q^*_1 t = z_1 t \beta_1 \gamma Q^*_1 t_{-1} \) increase or decrease at the same time without violating Equation (46).