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Abstract

We study minimal conditions for competitive behavior with few agents, adapting the strategic market game of Dubey (1982), Simon (1984) and Benassy (1986) to an indivisible good environment. We show that all Nash equilibrium outcomes with active trading are competitive if and only if there are at least two buyers and two sellers willing to trade at every competitive price. Unlike previous formulations, this condition can be verified directly by checking the set of competitive equilibria. In laboratory experiments, the condition we provide turns out to be enough to induce competitive results. Moreover, the performance of a sealed-bid auction following the rules of the strategic market game approaches that of its dynamic counterpart, the double auction, over time.
Keywords: market game, market experiment, double auction, perfect competition

1 Introduction

Ever since the classic contributions of Cournot (1838) and Bertrand (1883), the question of whether a market with a small number of traders can achieve competitive outcomes has been a matter of debate. The modern literature on strategic market games, pioneered by Dubey and Shubik (1980), revisits this topic in an environment in which buyers and sellers submit price-quantity pairs to a clearing house, which acts as a profit-maximizing middleman, and allocates trades accordingly. In line with Bertrand's argument, Dubey (1982), Simon (1984) and Benassy (1986) prove that having two active sellers and two active buyers in a Nash equilibrium is sufficient to make the outcome competitive.

In this study, we propose a strategic market game applicable to markets with indivisible commodities, we derive conditions for the equivalence between Nash equilibrium and competitive equilibrium outcomes, and we test the equivalence in the lab.

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We provide a novel necessary and sufficient condition for equivalence. Essentially, our condition requires that on each side of the market there are two inframarginal traders, in the sense that they are willing to trade at every competitive price.[1] Unlike previous work, our condition relies on the characteristics of the set of competitive equilibria, and place no requirement on Nash equilibria other than the occurrence of trade. Notably, our equivalence result includes contestable markets, in which a single active seller sells in the market at the competitive price.[2]

To test our results in the lab, we conduct market experiments with two buyers and two sellers—the minimal size allowing for the equivalence of Nash equilibrium and competitive equilibrium outcomes, and thus adequate for a stringent test. We consider two market environments: one in which the two buyers and the two sellers are inframarginal, so that all Nash equilibrium outcomes are competitive, and one in which the two buyers but only one of the sellers are inframarginal (i.e. there is monopoly power) so that some Nash equilibrium outcomes are non-competitive. In each environment, we consider two market institutions: a sealed-bid auction and a double-oral auction (following the rules of Smith (1962)), which are static/dynamic versions of each other.

In our laboratory experiments, as in other market experiments, traders are informed about their own valuations but not about the valuations of other traders. Thus, in using a strategic market game to explain behavior in the lab, we are following what Friedman and Ostroy (1995) call the "as-if Nash equilibrium complete information approach," the underlying idea of which is that "although traders' information in the experiments is far from complete, it is possible for them to learn to use the relevant 'complete information' strategies" (p. 23). The double-oral auction institution is known to facilitate learning of the relevant information for traders when compared to call markets, with as few as eight traders (see e.g. Smith, 1982; Smith et al., 1982), and hence provides a useful benchmark for assessing the equivalence result.

In the absence of monopoly power, the results from our experiment confirm the double auction institution's convergence to competitive outcomes, though we have fewer traders than previous experiments.[3] Efficiency under the sealed-bid institution remains below efficiency under the double-oral auction, but seemingly converges over time, in line with the results obtained by Smith et al. (1982) and Friedman and Ostroy (1995) for larger numbers of traders. Under both institutions, trading prices lay mostly in the competitive range in the absence of monopoly power, consistent with equilibrium predictions.

When monopoly power exists, higher trading price, lower trading volume and an efficiency loss can be observed under the double-oral auction compared to the environment without monopoly, as expected. Under the sealed-bid institution, trading volume is lower compared to the environment without monopoly, but the efficiency loss is not significant, and prices seem to converge to competitive levels over time. This surprising result may be either a consequence of the inability of the monopolist to gather enough information about the other side of the market to exploit monopoly power under the

[1] Our exact condition, spelled precisely in the statement of theorem 3, is slightly weaker.

[2] To prove our equivalence result, we first extend results from previous literature to our indivisible commodity setting in theorems 1 and 2. The proof of theorem 3 builds on those results and handles the additional contestable market case.

[3] See figure 7 for a comparison between efficiency in our experiment and others in the literature.

sealed-bid institution, or a consequence of coordination on a low-price outcome, which remains a Nash equilibrium outcome under monopolistic conditions. It is an interesting and open question whether the convergence to competitive outcomes for the sealed-bid institution even in the presence of monopoly power is robust to learning with a longer horizon and to variations in the parameters describing the economy.

We explore two other issues related to efficiency and learning suggested by previous experimental and theoretical literature. Plott et al. (2013) provides some experimental evidence that, before the market settles in equilibrium prices, efficiency in trading using the double-oral auction may be helped by a "Marshallian path," i.e. trading occurring first those who have more to gain from trade. Accordingly, we expect trading in the competitive environment under the double-oral auction to occur first among the buyer and the seller with respectively the lower cost and the higher value for the good. This conjecture is confirmed by our data. In particular, we observe the Marshallian path to hold better in earlier rounds, as corresponds to a phenomenon linked to market learning out of equilibrium.

Finally, we consider whether exploiting monopoly power under the double-oral auction institution is hindered by "Coasian dynamics." In our experiment, Coasian dynamics would prevent the monopolist seller to sell to the highest valuation buyer at above competitive prices because that buyer would anticipate that the monopolist would be willing to lower the price afterwards to trade with the other buyer. We do find some evidence of the monopolist occasionally lowering the price to sell a second unit. However, trading volume remains in average below competitive levels, and estimated price asymptotes suggest higher than competitive prices in the monopolistic environment under the double-oral auction.

The rest of the paper is organized as follows. Section 2 gives a formal description of the economy. Section 3 gives a detailed explanation of the strategic market mechanism. Section 4 contains the theorems of coincidence of Nash equilibrium and competitive equilibrium. Section 5 presents the experimental design and hypotheses. Section 6 describes the results. Section 7 concludes. Proofs for the main results are collected in the appendix, and additional proofs, graphs, and experimental instructions and quizzes are collected in the online appendix.

2 The economy

We describe a general equilibrium model related to laboratory experiments. Our notation follows Friedman and Ostroy (1995). There are two goods, a divisible 'money' and a traded good that can only be traded in indivisible units. Let $I = B \cup S$ be the set of individuals, classified as either buyers (B) or sellers (S). Each $i \in I$ is defined by a vector $(r_{i1}, \ldots, r_{ik})$, where r_{ij} indicates the reservation value for the j^{th} unit of the traded good. The parameter $k \geq 1$ indicates the maximum number of units of the traded good that an individual can buy or sell. For each $i \in B$, reservation values decrease with the quantity demanded: $r_{i1} \geq \cdots \geq r_{ik} \geq 0$. For each $i \in S$, reservation values increase with the quantity supplied $0 \leq r_{i1} \leq \cdots \leq r_{ik}$.

Each trader's utility is given by

$$u_i(q_i,m_i) = \begin{cases} \delta_i \sum_{j=1}^{|q_i|} r_{ij} + m_i & \text{if } q_i \neq 0 \\ m_i & \text{if } q_i = 0 \end{cases}, \quad \text{with} \quad \delta_i = \begin{cases} 1 & \text{if } i \in B \\ -1 & \text{if } i \in S \end{cases},$$

where $q_i \in Q_i$ is the quantity of the good traded by i and $m_i \in \Re$ are the money holdings of i. We let $Q_i = \{0,1,\ldots,k\}$ if $i \in B$ and $Q_i = \{0,-1,\ldots,-k\}$ if $i \in S$, so that supply is described as negative demand. We assume that initial endowment of money of each individual is equal to 0; note that individuals are allowed negative money holdings.

Keeping fixed the sets of buyers and sellers and k, an economy $r \in \Re_+^{k|I|}$ is described by a set of vectors of reservation values that are weakly decreasing for each buyer and weakly increasing for each seller, as described above. Given an economy r, an *allocation* (of the indivisible good) is a vector $q = (q_i) \in \times_{i \in I} Q_i$ and an *outcome* is a vector (q,m) where q is an allocation and $m \in \Re^{|I|}$.

Denote by $\xi(r)$ the set of competitive equilibria for an economy r. A *competitive equilibrium* $(p,q) \in \xi(r)$ is a price $p \in \Re_+$ and an allocation q such that

1. (utility maximization) for each i, $u_i(q_i,-pq_i) \geq u_i(q_i',-pq_i')$ for all $q_i' \in Q_i$.
2. (market clearance) $\sum_{i \in I} q_i = 0$.

By utility maximization, if (p,q) is a competitive equilibrium for economy r, then

- for every $i \in B$, either $q_i = 0$ and $r_{i1} \leq p$, or $0 < q_i < k$ and $r_{iq_i} \geq p \geq r_{i,q_i+1}$, or $q_i = k$ and $r_{ik} \geq p$.
- for every $i \in S$, either $q_i = 0$ and $r_{i1} \geq p$, or $-k < q_i < 0$ and $r_{i|q_i|} \leq p \leq r_{i,|q_i|+1}$, or $q_i = -k$ and $r_{ik} \leq p$.

Note that (p,q) induces the outcome $(q,m) = (q,(-pq_i))$.

It is easy to prove that for any economy r, there is a competitive equilibrium. We can order the units that sellers can supply in ascending order according to their reservation values, and the units that buyers can demand in descending order according to their reservation values, to obtain the familiar supply and demand curves. Equilibrium prices and allocations can be obtained by the crossing of the supply and demand curves. As it is well-known for economies with quasi-linear preferences, the set of competitive allocations is the set of solutions to the problem of maximizing social surplus, that is

$$\max_{q \in Q} \sum_{i \in I} \sum_{0 \leq j \leq |q_i|} \delta_i r_{ij},$$

where $Q = \{q : q_i \in Q_i, \sum_i q_i = 0\}$ is the set of feasible allocations.

Trade is positive in every competitive equilibrium if and only if

$$\min_{i \in S} r_{i1} < \max_{i \in B} r_{i1}. \tag{A}$$

As we will see, an important condition for the equivalence between competitive equilibrium outcomes and the outcomes of a strategic game is that there are at least *two* trading individuals on each side of the market.

Related work on price-quantity strategic market games feature divisible commodities under the usual assumptions of continuous, increasing marginal costs for each seller, and continuous, decreasing marginal utility of consumption for each buyer. Note that in economies with divisible units active traders compete "at the margin," in the sense that in a competitive equilibrium the marginal utility of consumption and the marginal cost of production for the last unit are equated to the price for all active traders. Our main result illustrates that competition at the margin is unnecessary for the equivalence between competitive and strategic outcomes.

3 The strategic market game

Each individual submits a price-quantity offer $(\widetilde{p}_i, \widetilde{q}_i)$ to the clearing-house, where $\widetilde{p}_i \geq 0$ and $\widetilde{q}_i \in Q_i$. Intuitively, each individual offers to trade up to $|\widetilde{q}_i|$ units of the traded good at the price $\widetilde{p}_i$. Denote the set of feasible offers for individual i by

$$W_i = \{(\widetilde{p}_i, \widetilde{q}_i) : \widetilde{p}_i \geq 0;\ \widetilde{q}_i \in Q_i\}.$$

Given an offer profile $w \in W = \times_{i \in I} W_i$, the set of feasible allocation vectors for the clearing house is

$$\begin{aligned} Y(w) = \{(y_1, ... y_n)\colon & 0 \leq y_i \leq \widetilde{q}_i, \text{ if } i \in B; & (3.1)\\ & 0 \geq y_i \geq \widetilde{q}_i, \text{ if } i \in S; & (3.2)\\ & \textstyle\sum_i y_i = 0; & (3.3)\\ & y_i \in \mathbb{Z}\}. & (3.4) \end{aligned}$$

Note that $Y(w)$ is a finite set. Conditions (3.1) and (3.2) guarantee that trade is voluntary, i.e. individuals do not end up trading more than what they offered. Condition (3.3) ensures that the market clears and the clearing house keeps no inventory. Condition (3.4) conveys the assumption that the good is indivisible.

After the clearing house chooses an allocation $y = (y_1, \ldots, y_n) \in Y(w)$, individual i receives y_i units of the traded good and earns an amount of money equal to $-\widetilde{p}_i y_i$. We assume that the clearing house allocates trade to maximize the arbitrage profit, $\sum_{i \in I} y_i \widetilde{p}_i$, as if the clearing house buys units from the sellers and sells them to buyers at the agents' proposed prices. Thus, given an offer profile w, the resulting allocation y must satisfy

$$y \in \Pi(w) = \{y \in Y(w) : \sum_{i \in I} y_i \widetilde{p}_i \geq \sum_{i \in I} y'_i \widetilde{p}_i \text{ for all } y' \in Y(w)\}.$$

Intuitively, as in Dubey (1982), buying offers are ranked in a descending order by price while the quantities offered are accumulated to form the demand curve, and selling offers are ranked in an ascending order by price while the quantities offered are accumulated to form the supply curve. The clearing house extracts the surplus between the supply and demand, as Figure 1 illustrates. That is, the clearing house chooses a competitive equilibrium allocation for a fictitious economy $\tilde{r}$ given by

$$\widetilde{r}_{ij} = \begin{cases} \widetilde{p}_i & \text{if } 1 \leq j \leq |\widetilde{q}_i| \\ 0 & \text{if } |\widetilde{q}_i| < j \leq k \text{ and } i \in B \\ +\infty & \text{if } |\widetilde{q}_i| < j \leq k \text{ and } i \in S \end{cases},$$

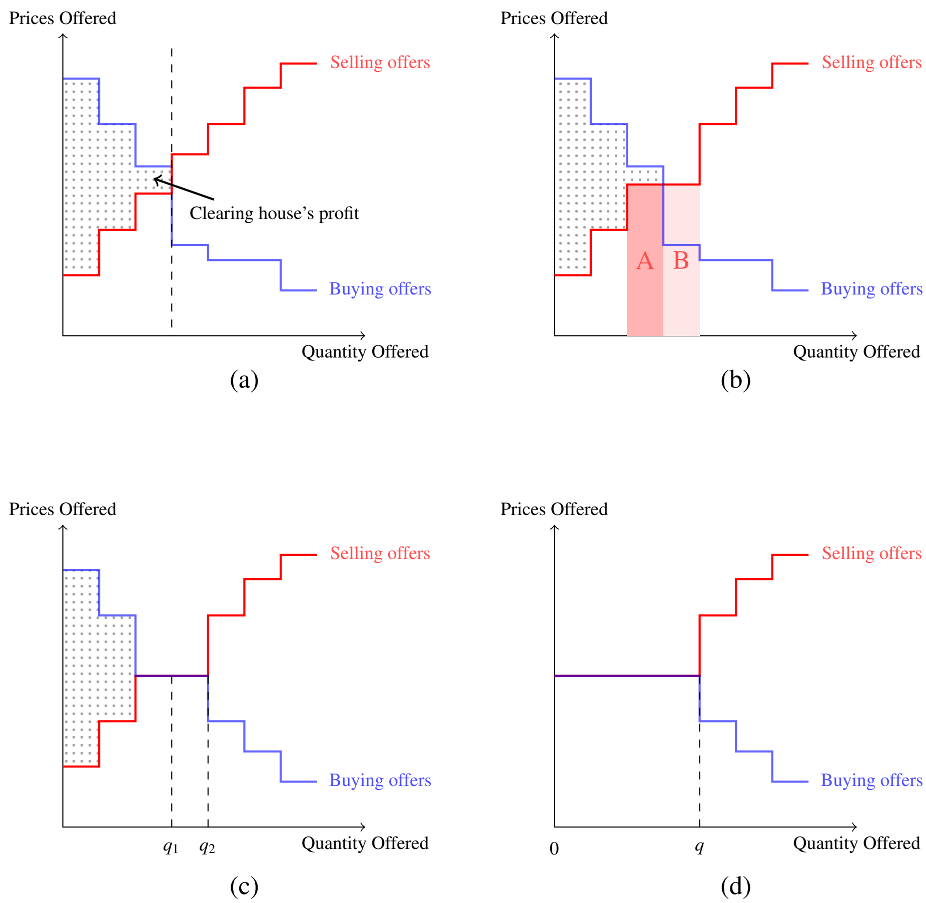


Figure 1: Arbitrage profit for the clearing house

and appropriates the social surplus.

In scenario (a) of Figure 1, $\Pi(w)$ is a singleton set. To maximize the arbitrage profit, the clearing house would fulfill all demand and supply to the left of the dashed line. The dotted area is the profit for the clearing house, and the profit is positive in this case. In scenario (b), $\Pi(w)$ contains two allocations if units A and B are offered by different sellers, depending on which of the two sellers is allowed to sell the last unit. In scenario (c), the clearing house gets the same profit allocating q_1 or $q_2 > q_1$ units. Similarly, in scenario (d), buying and selling q units gives the same profit for the clearing house as making no trade.

To make trade happen whenever possible, following Simon (1984), we assume that the clearing house chooses an allocation from the set

$$F(w) = \{y \in \Pi(w) : \text{there is no } \phi \in \Pi(w) \text{ such that } \phi \neq y \text{ and } |\phi_i| \geq |y_i| \text{ for all } i \in I\}.$$

That is, the clearing house does not choose allocations that are ray-dominated. Then in scenario (c), q_2 units will be bought and sold, and in scenario (d), q units will be traded. We still have two allocations in $F(w)$ in scenario (b) if units A and B are offered by different sellers. We assume that the clearing house chooses randomly according to the distribution μ_w that gives probability $\mu_w(y) > 0$ to each allocation $y \in F(w)$ and probability $\mu_w(y) = 0$ to every other allocation in $Y(w)$ such that $\sum_{y \in F(w)} \mu_w(y) = 1$. Propositions 1–6 in the Appendix provide a characterization on $F(w)$.

Given this market mechanism, define an *active trader* given offer profile w as a trader that has positive probability to trade. In other words, agent i is an *active trader* given offer profile w if there exists $y \in F(w)$ such that $y_i \neq 0$. Furthermore, denote by $AS(w)$ the set of active sellers, and $AB(w)$ the set of active buyers given offer profile w.

4 Nash equilibrium and competitive outcomes

Note that each offer profile $w \in W$ induces a lottery over outcomes. Each outcome $(y, (-\widetilde{p}_i y_i))$ is realized with probability $\mu_w(y) > 0$ if $y \in F(w)$, and $\mu_w(y) = 0$ if not. Given an offer profile $w \in W$, the expected utility for each trader is,

$$Eu_i(w) = \sum_{y \in F(w)} \mu_w(y) u_i(y_i, -\widetilde{p}_i y_i).$$

A (pure strategy) *Nash equilibrium* for an economy r is an offer profile $w^* \in W$ such that for every $i \in I$,

$$Eu_i(w_i^*, w_{-i}^*) \geq Eu_i(w_i', w_{-i}^*) \text{ for all } w_i' \in W_i.$$

As in other price-quantity strategic market games, every competitive equilibrium outcome can be reached by with probability one by at least one Nash equilibrium offer profile, and all the positive probability outcomes of a Nash equilibrium are competitive as long as in the Nash equilibrium there are at least two active traders on each side of the market.

Theorem 1. *For every competitive equilibrium, there is a Nash equilibrium that induces the same outcome with probability one.*

To prove the theorem, we consider an offer profile such that each agent offers the trading price and quantity she obtains in the competitive equilibrium, and show that such offer profile is a Nash equilibrium and yields exactly the same outcome as in the competitive equilibrium. Agents have no incentive to deviate from the proposed offer profile: since the quantity offered in the profile is utility-maximizing given the competitive price, obtaining a different quantity at the same price does not increase the payoff for the individual; given other agents are offering the same price, increasing offer price as a seller or decreasing offer price as a buyer, regardless of the quantity offered, reduce the chance of trade to 0, and thus cannot be payoff-improving; decreasing offer price as a seller or increasing offer price as a buyer reduces the payoff for sure as the new price is less preferred to the competitive price, even at its corresponding utility-maximizing quantity.

As long as condition (A) is satisfied (which is, of course, the case of interest), there are Nash equilibria that induce noncompetitive allocations. For instance, any offer profile such that $\widetilde{q}_i = 0$ for all i, or such that $\min_{i\in S}\widetilde{p}_i > \max_{h\in B} r_{h1}$ and $\max_{i\in B}\widetilde{p}_i < \min_{h\in S} r_{h1}$, is a Nash equilibrium. Those Nash equilibria result in no trade. We restrict our attention on Nash equilibria such that trade happens with positive probability, so that $AS(w)$ and $AB(w)$ are nonempty. We have

Theorem 2. *In every Nash equilibrium with at least two active traders on each side, every positive probability outcome is competitive.*

To prove theorem 2, we first show that in any given Nash equilibrium, all active traders offer the same price. Then we show that in every allocation induced by a Nash profile, the quantity that an active trader is allocated is utility-maximizing given the Nash price. The intuition is that if an active buyer/seller does not get the utility-maximizing quantity at the Nash price, the buyer/seller can always obtain a more preferable quantity by offering a slightly higher/lower price.

Note that there is a gap between the statement of theorem 2 and the no-trade examples preceding the statement of the theorem. Theorem 2 leaves open the possibility that there are Nash equilibria with active trading but with noncompetitive outcomes and in which there is only one active trader in at least one of the two sides of the market. In the proof of the theorem, we rely on two or more active sellers in order to show that there is no Nash equilibrium in which one seller produces less than the competitive allocation requires. Intuitively, these situations would correspond to the single active seller behaving as a monopolist and charging a price above the competitive level. Similarly, there could be situations in which there is a single active buyer behaving as a monopsonist and charging a price below the competitive level. Finally, there could be situations in which there is a single active buyer and a single active seller, and competitive outcomes are not reached even if the price is competitive because of a coordination failure: both the buyer and the seller offer suboptimal quantities.

In what follows, we provide necessary and sufficient conditions for all the outcomes of *every* Nash equilibrium with trade to be competitive. Define the *buyers' marginal value*, v_b, as the maximum of the lowest reservation value for buyers' units traded in competitive equilibria, that is,

$$v_b = \max_{(p,q)\in\xi(r)} \min_{q_i>0} r_{i,q_i}.$$

Similarly, define the *sellers' marginal value*, v_s, as the minimum of the highest reservation value for sellers' units traded in competitive equilibria, that is,

$$v_s = \min_{(p,q)\in\xi(r)} \max_{q_i<0} r_{i,|q_i|}.$$

In economies such that (A) is satisfied, v_b and v_s are well-defined, since in every competitive equilibrium at least some $i' \in S$ with the minimum cost (i.e. $r_{i'1} =$

$\min_{i\in S} r_{i1}$) must have $q_{i'} < 0$, and at least some $i'' \in B$ with the maximum reservation value (i.e. $r_{i''1} = \max_{i\in B} r_{i1}$) must have $q_{i''} > 0$. As shown in the Appendix, v_b and v_s are equal, respectively, to the lowest reservation value of buyers' traded unit(s) and the highest reservation value of sellers' traded unit(s) in any competitive equilibrium with the smallest number of transactions. Moreover, if (A) is satisfied, we must have $v_b > v_s$, because if there is a competitive equilibrium such that both the marginal buyer and the marginal seller are indifferent (i.e. $\min_{q_i>0} r_{i,q_i} = p = \max_{q_i<0} r_{i,|q_i|}$), there is another competitive equilibrium in which one fewer unit is traded.

Denote by $\overline{p}$ and $\underline{p}$ the highest and lowest competitive price respectively. It is easy to check that

$$v_s \leq \underline{p} \leq \overline{p} \leq v_b.$$

The first and third inequalities above follow from the fact that for every equilibrium $(p,q) \in \xi(r)$ we must have $\max_{q_i<0} r_{i,|q_i|} \leq p \leq \min_{q_i>0} r_{i,q_i}$.

We say that $i \in B$ is an *inframarginal buyer* if $r_{i1} \geq v_b$. Similarly, we say that $i \in S$ is an *inframarginal seller* if $r_{i1} \leq v_s$. Intuitively, an inframarginal trader is someone who is willing to trade at every competitive equilibrium price. Note that in economies satisfying (A), there is at least one inframarginal trader on each side of the market, since every seller with the minimum cost and every buyer with the maximum reservation value is inframarginal.

We say that $i \in B$ is a *weakly inframarginal buyer* if $r_{i1} > v_s$ and $r_{i1} \geq \underline{p}$. Similarly, we say that $i \in S$ is a *weakly inframarginal seller* if $r_{i1} < v_b$ and $r_{i1} \leq \overline{p}$. Intuitively, a weakly inframarginal trader is someone who would generate positive social surplus if matched in pairwise trade with an inframarginal trader on the other side of the market. Using $v_b > v_s$ and $v_s \leq \underline{p} \leq \overline{p} \leq v_b$, it is easy to check that, in economies satisfying (A), all inframarginal traders are also weakly inframarginal (justifying our nomenclature).

If an economy has competitive equilibria in which only one unit is traded, then all outcomes of every Nash equilibrium profile with trade are efficient.[4] From here on, we focus on economies such that all competitive equilibria involve trading two or more units, which is a more demanding condition than (A).

We have

Theorem 3. *In economies such that all competitive equilibria involve trading two or more units, every positive probability outcome from every Nash equilibrium with active trade is competitive if and only if there are at least two inframarginal traders on one side of the market, and at least two weakly inframarginal traders on the other side.*

Intuitively, rivalry between two traders on the same side of the market who can exploit mutually advantageous trades with at least two traders on the other side of the market both eliminates monopoly and monopsony power and precludes coordination failures. In the coordination failure example proposed above, we have $v_s = 1$, $v_b = 3$,

[4] The reason is that if one unit is traded in a given outcome induced by an equilibrium profile, both the active buyer and the active seller must be offering the same price. If any buyer has a reservation price higher than the Nash price and is not trading, the buyer can offer a price that is slightly higher and grab the trade, so that in equilibrium the buyer who trades must be the one with the highest reservation price. A similar argument applies on the supply side. (However, the trading price may not be competitive in the Nash equilibrium.)

and all traders are weakly inframarginal but only one seller and one buyer are inframarginal.

The condition $r_{i1} \geq \underline{p}$ for $i \in B$ and $r_{i1} \leq \overline{p}$ for $i \in S$ to be a weakly inframarginal trader ensures that the trader has a value "close enough" to the competitive range, so that the trader weakly prefers to trade in the competitive equilibrium. Without this condition, there may be noncompetitive Nash equilibrium outcomes. Consider the economy $S = \{1,2\}$, $B = \{3,4\}$, $k = 3$, and $r_{11} = r_{21} = 1$, $r_{12} = r_{13} = r_{22} = r_{23} = 4$, $r_{31} = r_{32} = r_{33} = 3$, $r_{41} = r_{42} = r_{43} = 1$. Seller 1, seller 2, and buyer 3 are inframarginal traders, while buyer 4 satisfies one of the conditions to be weakly inframarginal but not $r_{i1} \geq \underline{p}$. Here buyer 4 strictly prefers not to trade in the competitive equilibrium, and the range for Nash equilibrium prices is $[2,3]$, including prices that are not competitive.

It is worth noticing that theorem 3 includes the contestable market scenario (Baumol et al., 1982), in which there is only one active seller but all outcomes from Nash equilibria are competitive. An example is the economy $S = \{1,2\}$, $B = \{3,4\}$, $k = 2$, and $r_{11} = r_{12} = r_{21} = r_{22} = 2$, $r_{31} = 4$, $r_{41} = 3$, $r_{32} = r_{42} = 1$. The competitive equilibrium price is 2 in this economy, and two units are traded in every competitive equilibrium. We have $v_b = 3$ and $v_s = 2$ for this economy, so all traders are inframarginal and the condition in theorem 3 holds. One of the Nash equilibria in this economy is $w = ((2,-2),(2,0),(2,1),(2,1))$, in which seller 1 is the only active seller, but the outcome is competitive. The presence of seller 2, a non-active seller in the Nash equilibrium, brings enough competition to the market to make the outcome competitive.

5 Experimental design and hypotheses

5.1 Experimental design

We test the predictive ability of our market game model in laboratory experiments. We consider two markets with indivisible commodities. Each market has two buyers $B = \{B1, B2\}$ and two sellers $S = \{S1, S2\}$, and each trader can either buy or sell two units. We assign the first and third highest demand reservation values to one buyer, and the second and fourth to the other buyer. By assigning the units to sellers in different ways, we create a market that satisfies the condition in theorem 3, and a market that does not. This design is similar to one implemented by Davis and Holt (1994).

In our competitive market, the two supply units that can be traded in competitive equilibrium are assigned each to each one of the two sellers. Thus, as shown in the left part of figure 2, there are two inframarginal traders on each side of the market. By theorem 3, Nash equilibrium outcomes with trading of the strategic market mechanism coincide with competitive equilibrium outcomes. That is, both units with lowers costs should be traded, and the price should be in the competitive price range, \$15–\$19. Correspondingly, efficiency (as percentage of the maximum possible surplus) should be 100%.

In our monopoly market, instead, the two low cost units are assigned to the same seller, as shown in the right part of figure 2. The set of Nash equilibrium outcomes with trade includes the set of competitive equilibria just described, as well as monopolistic market equilibria in which only the unit with the lowest cost is traded, and the price is

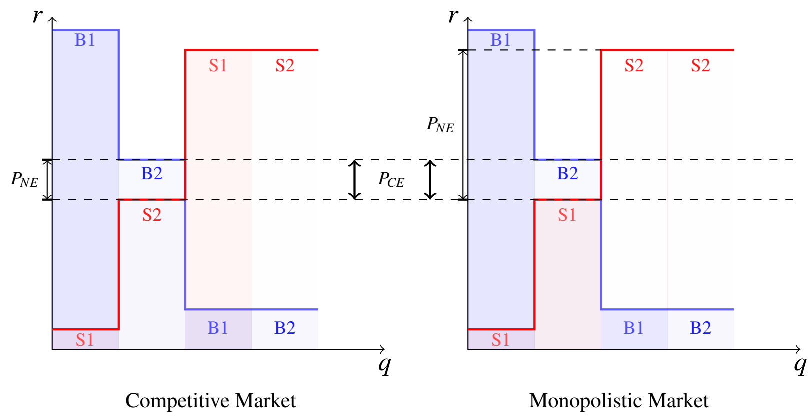


Figure 2: Competitive and monopolistic markets

between \$19 and \$30. Efficiency of monopolistic equilibria is $(32-2)/(32-2+19-15)$, that is approximately 88%.

In the experiment, we compare the performance of a sealed-bid institution (Clearing House, or CH hereafter) with a continuous time double auction (Double Auction, or DA hereafter) in the two markets.

In the clearing house institution, each trader submits a price-quantity pair to the clearing house. The clearing house then decides trade by the rules described in section 3, and reports the trader's own transaction price and quantity, together with the price and quantity traded in the market. We let $\mu_w(y) = 1/|F(w)|$ for all $y \in F(w)$ in the experiment; that is, the clearing house assigns equal probability to all arbitrage-maximizing allocations. When making decisions, traders are given their own values, but not other traders' values or offers.

In the double auction institution, the traders buy/sell the good unit by unit. Each trader can submit limit offers for one unit, and each limit offer has to reduce the bid-ask spread to be valid. Valid offers are listed on the screen as public information for all traders in the market, with bids ranked from high to low and asks ranked from low to high. A transaction happens automatically if a valid bid is no lower than a valid ask. In each transaction, a bid will always be matched with the highest-ranked ask, and an ask will be traded with the highest-ranked valid bid. The trading price will be the price in the pair that was submitted later. After a trader has the first unit traded, he/she can submit limit offers for the second unit. All valid offers and transactions are shown in real time to all traders in the market.

The experiment was conducted in the Interdisciplinary Center for Economic Science (ICES) lab in George Mason University. In total, 240 subjects participated in the 18 sessions, and each session lasted for no more than 100 minutes. Each subject participated in only one treatment, playing the same role (B1, B2, S1 or S2) in the same market for 20 rounds. The final payoffs ranged from \$5 to \$36.

The experiment was computerized, and programmed in oTree (Chen et al., 2016). At the beginning of the session, the participants were seated at partitioned computer stations and allowed 10 minutes to read the instructions on their own. Then an experimenter read the instructions out loud to all participants. Afterward, a quiz was handed out, and the experiment began after each participant gave correct answers to all the questions in the quiz. Then the role a participant had in the experiment was revealed to him/her, and the participants were given a practice round before the formal rounds began. There were 20 formal rounds, one of which was randomly chosen for payment. After the 20 formal rounds, each participant was informed of the round chosen for payment and his/her own payoff. The payment was made privately.

5.2 Hypotheses

Our first set of empirical hypotheses correspond to treatment effects. Because the set of equilibria under monopolistic conditions includes inefficient outcomes with prices above competitive levels, we expect treatments with competitive markets to exhibit lower prices and higher efficiency, together with higher trading volume, higher surplus for buyers and lower surplus for sellers. And given the advantage for learning of the double auction institution over the clearing-house, we expect treatments under the double auction institution to exhibit higher efficiency.

(H1) Under the double auction institution, prices and sellers' total surplus are lower, and efficiency, trading volume and buyers' total surplus are higher, in competitive markets than in monopolistic markets.

(H2) Under the clearing-house institution, prices and sellers' total surplus are lower, and efficiency, trading volume and buyers' total surplus are higher, in competitive markets than in monopolistic markets.

(H3) In competitive markets, efficiency under the double auction institution is higher than under the clearing-house institution.

(H4) In monopolistic markets, efficiency under the double auction institution is higher than under the clearing-house institution.

Our second set of empirical hypotheses correspond to the convergence to competitive prices in the long run if the market has a competitive structure under both institutions.

(H5) Under the double auction institution, prices converge to the competitive range in competitive markets.

(H6) Under the clearing house institution, prices converge to the competitive range in competitive markets.

Our next set of empirical hypotheses correspond to predictive success of equilibrium notions. Because the set of equilibria under monopolistic conditions is a strict superset of the set of equilibria under competitive conditions, we expect competitive predictions to perform better in the latter case. And given the advantage for learning of the double-oral auction over the clearing-house, we expect Nash predictions to perform better in in the former case.

(H7) Competitive predictions perform better in competitive markets than in monopolistic market under the double-oral auction.

(H8) Competitive predictions perform better in competitive markets than in monopolistic markets under the clearing-house auction.

(H9) Competitive predictions perform better under the double-oral auction than under the clearing-house in competitive markets.

(H10) Nash predictions perform better under the double-oral auction than under the clearing-house in monopolistic markets.

Our next hypothesis is concerned with adjustment toward equilibrium and learning under the double-oral auction. In particular, previous experimental work (Cason and Friedman, 1996; Plott et al., 2013) points to a Marshallian path during price convergence to equilibrium levels, which helps to achieve efficient trading even if prices are out of equilibrium.

(H11) In competitive markets, under the double-oral auction, the first traded unit is the lowest cost, it is bought by the highest valuation buyer, and in consequence it generates more surplus than the second traded unit.

Finally, we consider the possibility of Coasian dynamics. The monopoly under the double auction institution in our experiment faces a problem similar to the durable goods monopoly in Coase (1972). After selling one unit at above competitive prices, it is in the monopolist seller's best interest to lower the price and sell another unit. Expecting this, buyers may withhold purchases until the price is at a competitive level. Nonetheless, the seller could have extracted a higher profit if committing to sell only one unit at a high enough price. A weak version of Coasian dynamics is that during adjustment toward equilibrium, the monopolist should experiment offering the second unit at a lower price and thus be able to sell two units. A stronger version of Coasian dynamics should imply selection of a competitive outcome under the double auction even under monopolistic conditions.

(H12) In monopolistic markets, under the double-oral auction, two units are sold, and prices converge to the competitive range.

6 Results

6.1 Treatment effects

6.1.1 Overview

Table 1 presents treatment effects using the last ten rounds. Efficiency is defined as the percentage of the maximum social surplus realized. Trading volume is defined as the number of units traded divided by two (the number of inframarginal units), in percentage. Buyers' and sellers' surplus are defined as percentage of the maximum possible social surplus. In agreement with H1, under the double auction institution,

			Trading price	Efficiency (%) [a]	Trading volume (%) [b]	Buyers' surplus (%) [c]	Sellers' surplus (%) [c]
Mean values	CH Competitive	(17 markets)	15.73	69.45	58.53	36.32	30.81
			(2.70)	(32.39)	(28.80)	(21.33)	(17.62)
	CH Monopoly	(16 markets)	15.78	71.95	52.19	31.36	32.51
			(4.60)	(30.17)	(25.30)	(21.02)	(18.66)
	DA Competitive	(14 markets)	16.89	88.55	80.36	44.37	44.18
			(2.28)	(22.77)	(26.62)	(15.88)	(15.08)
	DA Monopoly	(13 markets)	18.18	75.07	57.31	33.06	42.00
			(5.41)	(33.57)	(31.81)	(21.27)	(24.05)
Kruskal-Wallis test			<.001	<.001	<.001	<.001	<.001
Mann-Whitney U test [d]	Competitive vs Monopoly	CH	.951[-]	.420[+]	.015[+]	.017[+]	.476[-]
		DA	<.001[-]	<.001[+]	<.001[+]	<.001[+]	.290[-]
	Clearing House vs Double Auction	Competitive	<.001[-]	<.001[-]	<.001[-]	.008[-]	<.001[-]
		Monopoly	<.001[-]	.011[-]	.051[-]	.722[-]	<.001[-]

Note: Data includes only the last 10 rounds. Standard deviations in parentheses. p-values are reported for statistical tests.
[a] Percentage of maximum social surplus realized. Efficiency = sellers' surplus + buyers' surplus + clearing house's profit.
[b] Number of unit traded in the market over two – the number of units traded in competitive equilibrium.
[c] As a percentage of maximum social surplus.
[d] [+] and [−] signs indicate the tested hypothesis being the difference is positive or negative.

Table 1: Treatment effects

trading prices are lower, and efficiency, trading volume, and buyers' are significantly higher in competitive markets than in monopolistic markets. Average sellers' surplus is higher in monopolistic markets but the difference is not significant. In agreement with H2, under the clearing-house institution, trading volume and buyer's surplus are significantly higher in competitive markets than in monopolistic markets. Differences in trading prices, efficiency, and sellers' surplus are not significant. In agreement with H3, in competitive markets, efficiency, trading volume, buyers' surplus, and sellers' surplus are higher under the double auction than under the clearing house institution. In agreement with H4, in monopolistic markets, efficiency, trading volume, and sellers' surplus are higher under the double auction than under the clearing house institution. Average buyers' surplus is higher under the double auction but the difference is not significant. Summing up, there is significant evidence in favor of H1 and H3, and some evidence in favor of H2 and H4.

6.1.2 Prices

Figure 3 shows average trading prices in each round in the four treatments. Two inferences can be drawn from figure 3. First, average prices adjust over time and stay in the competitive price range in the second half of the experiment in all treatments (average trading prices range from \$15.73 to \$18.18). The learning process takes longer under the clearing-house institution: the average price starts low, and reaches the competitive range over time. The upward sloping trend is not as strong under the double auction institution: the average trading price starts within the competitive range. Second, compared to competitive markets, monopolistic markets bring forth a higher average trading price under the double auction institution, but not so clearly under the clearing-house institution.

6.1.3 Efficiency

Figure 4 plots the average efficiency in each round in the four treatments. Efficiency is defined as the percentage of the maximum social surplus realized. Similar to what is shown in figure 3, learning takes longer under the clearing-house institution; hence, average efficiency under the clearing-house institution presents a stronger upward trend over time. Under the clearing-house institution, the average efficiencies start at levels lower than under the double auction institution, and remain statistically lower in the second half of the experiment. Nevertheless, we can observe from figure 3 that the upward trend of the efficiencies in clearing-house treatments persist over time, and at the end of the experiment, the efficiency levels from the two institutions are close.

6.1.4 Trading volume

In our setting, supramarginal trade occurs if a seller sells a unit with a cost of 30, or if a buyer buys a unit with a valuation of 4. In our experiment, in the last ten rounds, supramarginal trade occurred in 3 out of 199 trades in the CH Competitive treatment and in 4 out of 149 trades in the DA Monopoly treatment, and did not occur in other treatments. Thus, trading volume reflects inframarginal trading. Figure 5 and table

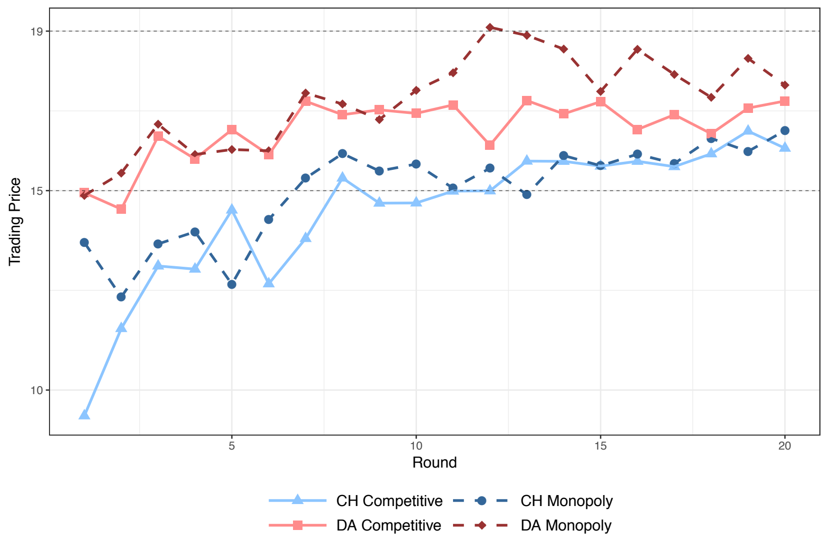


Figure 3: Average trading price

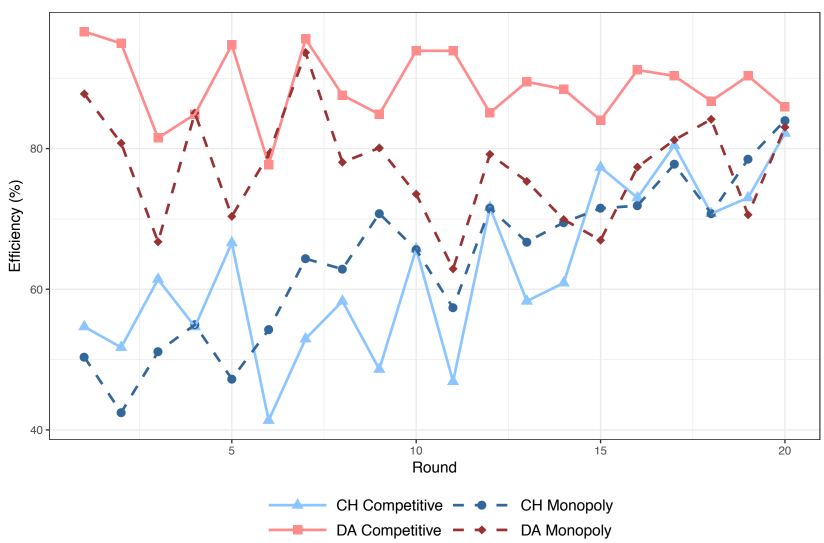


Figure 4: Average efficiency

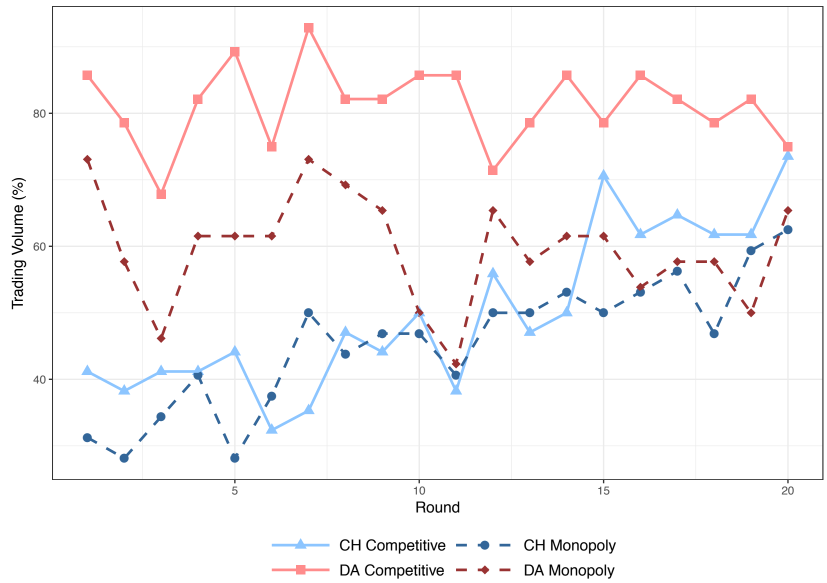


Figure 5: Average trading volume

1 illustrate that in the second half of the experiment, under both institutions, there are fewer trades in the monopolistic markets than in the competitive markets. Under the double auction, lower trading in monopolistic markets explains the advantage of competitive markets in terms of social efficiency and corroborates our hypotheses H1 and H2. As figure 5 and table 1 show, the clearing-house institution results in less trade than the double auction institution, corroborating our hypotheses H3 and H4.

6.2 Convergence to competitive prices

Following Noussair et al. (1995), we estimate

$$p_{it} = \alpha_1 D_1 \frac{1}{t} + \ldots + \alpha_i D_i \frac{1}{t} + \ldots + \beta \frac{t-1}{t} + \varepsilon_{it}$$

for each treatment, where p_{it} is the average price in market i at round t, D_i is an indicator for a specific market, which equals 1 if the market is i and 0 otherwise, β is the asymptote for the average price in the treatment, and ε_{it} is an error term. In using this statistical model, we assume that although each market has its own pattern of convergence, there is a common asymptote by treatment.

Table 2 lists the estimated β for each treatment. For competitive markets, the 95% confidence interval for long run prices is contained in the competitive price range under both the double auction and the clearing-house institution, providing corroborating support for H5 and H6. For monopolistic markets, the 95% confidence interval for long run prices is contained in the competitive price range for the clearing-house institution

Treatment	$\hat{p}^*$	95% Confidence interval
CH Competitive	16.12	$[15.68, 16.57]$
CH Monopoly	16.02	$[15.90, 16.14]$
DA Competitive	16.98	$[16.65, 17.32]$
DA Monopoly	19.20	$[18.51, 19.90]$

Feasible generalized LS estimation with AR(1) correction.

Table 2: Average price asymptote

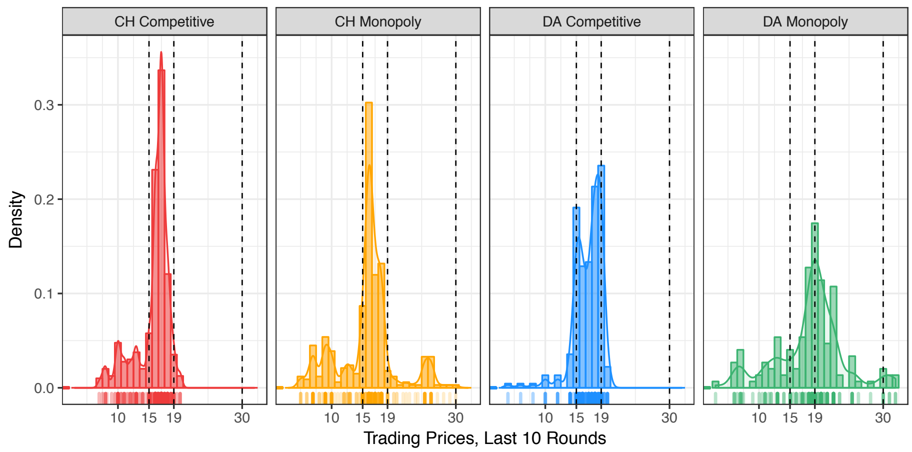


Figure 6: Distribution of trading prices

but not for the double auction. In fact, the confidence intervals are nested under the clearing-house institution but are disjoint under the double auction.

Figure 6 shows the distribution of trading prices in the last 10 rounds in different treatments. The DA Monopoly treatment has a heavy right tail outside of the competitive price range but within the Nash equilibrium range for that environment. In the CH Monopoly treatment, instead, most of the trading price within the Nash equilibrium range is also in the competitive price range. In both competitive treatments, trading prices cluster in the competitive price range.

Except for the DA Competitive treatment, all treatments have a heavy left tail. The heavy left tail may be due to slow learning, due to (i) lack of within-round feedback under the clearing-house institution, and (ii) less experimentation about possible prices when there is only one rather than two inframarginal sellers. Prices below the competitive equilibrium level were also observed by Smith and Williams (1990) in two monopolistic markets, perhaps for a similar reason.[5]

[5]Figures A1 – 4 in the online appendix plot all the offers under \$45 by round in each treatment.

	Predicted price range	
	Competitive ($15–$19)	Competitive + Monopolistic ($15–$30)
CH Competitive	**63.81%**	28.89%
DA Competitive	**75.11%**	42.44%
CH Monopoly	52.54%	**26.35%**
DA Monopoly	26.27%	**26.51%**

Table 3: Predictive success index

6.3 Predictive success

To explore whether Nash equilibrium is a good predictor for the experimental results, we use Selten's (1991) predictive success index. Selten's index is defined as the difference between the 'hit rate' (the percentage of data that is coherent with the prediction of the model) and the 'area' (the percentage of all possible outcomes that is coherent with the prediction of the model). Nash equilibrium predicts the range of competitive prices ($15–$19) for the competitive environment, and the range including both competitive prices and monopolistic prices ($15-$30) for the monopolistic environment. Given that participants cannot submit a price that may cause a loss, the possible price range in our experiment is $2–$32. Thus, the area equals 13.33% for the competitive range, and 50% for the combination of competitive and monopolistic price ranges.

Tables 3 summarizes the predictive success index of the two price ranges. The indices for Nash equilibrium are in bold. In agreement with our hypotheses H7 and H8, the competitive price range is a better prediction for competitive markets than in monopolistic markets, under both institutions and regardless of the index. In agreement with hypothesis H9, the competitive price range is a better prediction for competitive markets under the double auction than under the clearing-house institution. Opposite to hypothesis H10, Nash predictions perform similarly in monopolistic markets under both institutions. In fact, under the clearing-house institution, the competitive price range predicts better than the Nash range for monopolistic markets. Overall, predictive success indices indicate that learning to play equilibrium happens more easily in competitive markets, especially under the dynamic institution.

6.4 Marshallian path

We test the Spearman's rank correlation between transaction order and surplus from trade, buyer's value, and seller's cost respectively for our competitive markets under double auction institution. According to hypothesis H11, surplus from trade is higher, buyer's value is higher, and seller's cost is lower in the earlier transactions. The results from table 4 are in line with this prediction, except that in the second half of the experiment, buyers might have learned enough about equilibrium prices, and the effect of Marshallian path is no longer significant for buyer's value.

	Surplus from trade	Buyer's value	Seller's cost
Rounds 1–10	-0.39	-0.20	0.36
	($<.001$)	(.002)	($<.001$)
Rounds 11–20	-0.25	-0.10	0.23
	($<.001$)	(.148)	($<.001$)

Note: p-values in parentheses.

Table 4: Spearman's rank correlation coefficient (ρ) between transaction order and other variables

6.5 Coasian dynamics

The monopoly ability to exercise market power varies in double auction markets in laboratory experiments (Smith, 1981; Smith and Williams, 1990). Cason and Sharma (2001) find corroborating experimental evidence for Coasian dynamics in a two-period setting in which a monopoly sells durable goods to two buyers, each of whom has a privately known value for one unit. To see if Coasian dynamics is present in our data, we check whether the second transaction price in a round is lower than the first one, and whether the monopoly sells more than one unit. The MWW test for whether the second transaction price is lower has a p-value of .050, on the edge of rejecting the null hypothesis that trading prices are at the same level. The one-sided Wilcoxon signed rank test for whether more than one unit is sold yields the p-value of .005, rejecting the null hypothesis that the trading volume is 50% in the DA monopolistic market. Therefore, there is some evidence of Coasian dynamics in our double auction experiments. The estimated price asymptote, however, is not competitive in DA monopolistic markets. Hence the evidence for our hypothesis H12 is mixed.

7 Conclusions

In this paper, we aim to fill a gap in the theoretical and experimental literature about markets with few participants and indivisible commodities. First, we provide a necessary and sufficient condition for the equivalence of Nash equilibria of price-quantity strategic market games and competitive equilibrium outcomes. Second, we conduct market experiments in a competitive environment and in a monopolistic environment. We consider two market institutions, a sealed-bid auction (call market), following closely the rules of the market game, and a double-oral auction, which has been known to be successful in inducing competitive outcomes and prices in the lab.

Our lab experiments involve the minimum number of traders using the double auction that we know of. Figure 7 compares the efficiency level in our double auction markets with a few double auction markets in previous studies (Friedman and Ostroy, 1995; Kachelmeier and Shehata, 1992; Kimbrough and Smyth, 2018; Smith, 1982; Smith and Williams, 1990; Smith et al., 1982). Double auction markets conducted in previous studies are mostly used for testing the robustness of the mechanism, so disturbances may have been introduced during the session, and different settings have been

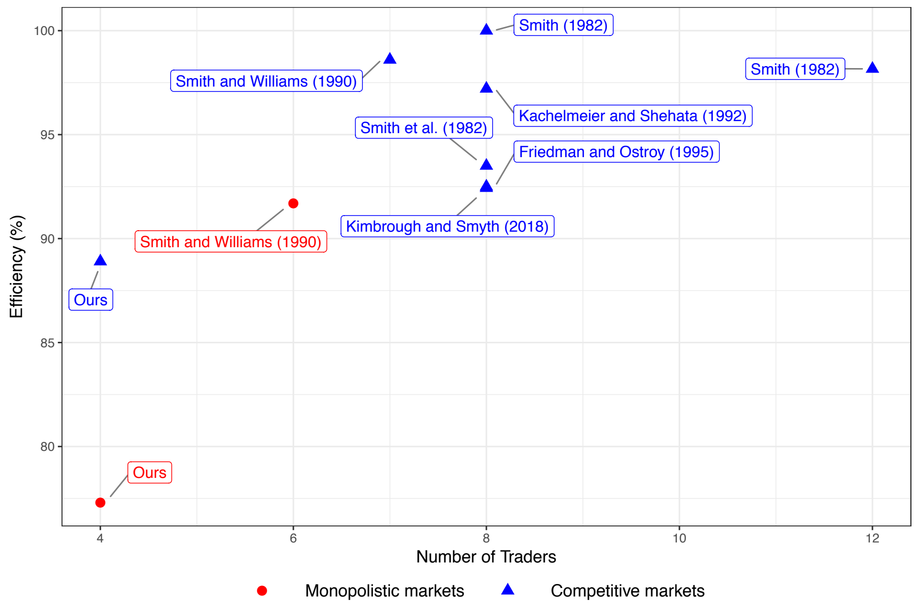


Figure 7: Efficiency and number of traders in the double-oral auction literature. All the points are average efficiency using all rounds.

used in these studies. Efficiency in thicker markets is higher than in our four-trader market, although the difference is not large when markets are competitive.[6]

Under the call market institution, efficiency is below that under the double auction in our experimental competitive markets. We interpret the advantage of the double auction as a result of better opportunities for learning. Nevertheless, the efficiency of the call market increases over time and gets closer to the double auction institution as traders in the market gradually learn. The approximation to competitive equilibrium outcomes is obtained without traders' knowledge of others' values under both institutions. Our results provide supportive evidence for the Hayek hypothesis (Hayek, 1945; Smith, 1982) in a limit setting with few traders: using appropriate institutions, markets can work with very limited information. Under the call market institution, transaction prices are the only information revealed to each trader other than their own value. This information appears to be sufficient for achieving equilibrium outcomes, although it may take a few trials.

In our experimental monopolistic markets, buyers' surplus and trading volume remains below that in our experimental competitive markets under both the double auction and the call market. The loss of total surplus in monopolistic markets is significant under the double auction although not under the call market. Tantalizingly, under the

[6]Using a different (quantity strategic) market game, where traders retain market power, Duffy et al. (2011) obtain higher efficiency and more coherence to competitive behavior if there are more traders. Dufwenberg and Gneezy (2000) also obtain a beneficial effect of the number of traders in an experiment on Bertrand competition between firms.

call market, prices are not in average higher in monopolistic than in competitive markets. Whether these observations about long-run prices can be generalized is left as an open question. Generally, Nash predictions from our strategic model do much worse in monopolistic markets. Learning enough to behave as if possessing complete information is seemingly much harder in monopolistic than in competitive markets.

Appendix A: Proofs

Characteristics of $F(w)$

The following propositions characterize the set of allocations chosen by the clearing house with positive probability after the offer profile w, and are used in later proofs. Proofs of the propositions themselves are lengthy but elementary and are collected in the online appendix.

Proposition 1. *For any $y \in F(w)$, if $y_b > 0$ and $y_s < 0$ for a buyer b and a seller s, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

Proof. Suppose $\widetilde{p}_b < \widetilde{p}_s$. Consider an alternative allocation y' such that $y'_i = y_i$ if $i \neq b, s$ and $y'_b = y_b - 1$, $y'_s = y_s + 1$. Since $y \in F(w) \subseteq Y(w)$, we have $y' \in Y(w)$, and

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_s - \widetilde{p}_b) > \sum_{i \in I} y_i \widetilde{p}_i.$$

Then $y \notin F(w)$, contradicting the assumption.

□

Proposition 2. *Given an offer profile w and a buyer b and a seller s such that $\widetilde{p}_b \geq \widetilde{p}_s$, there cannot be an allocation $y \in F(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$.*

Proof. For a given offer profile w such that there is a buyer b and a seller s that $\widetilde{p}_b \geq \widetilde{p}_s$, suppose there is an allocation vector $y \in Y(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$. We can show that $y \notin F(w)$. Take an alternative allocation vector y', let $y'_i = y_i$ if $i \neq b, s$, and $y'_b = y_b + 1$, $y'_s = y_s - 1$. We have $y' \in Y(w)$. The arbitrage profit for the clearinghouse by allocating y' is

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_s) \geq \sum_{i \in I} y_i \widetilde{p}_i,$$

where the last term is the clearing house's profit if it allocates y. Therefore, if $\widetilde{p}_b > \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i < \sum_{i \in I} y'_i \widetilde{p}_i$ and $y \notin \Pi(w)$; and if $\widetilde{p}_b = \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i = \sum_{i \in I} y'_i \widetilde{p}_i$ but y is ray dominated by y'. Either way we have $y \notin F(w)$.

□

Proposition 3. *If, for a given offer profile w, seller a and seller b offer $\widetilde{p}_a < \widetilde{p}_b$, and seller b is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$. Symmetrically, if in a given offer profile w, buyer a and buyer b offer $\widetilde{p}_a > \widetilde{p}_b$, and buyer b is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$.*

Proof. We will show the proof for the sellers' case, since the buyers' case is symmetric. By definition, if seller b is an active trader, there exists an allocation $y^* \in F(w)$ such that $y^*_b < 0$. First we show that $y^*_a = \widetilde{q}_a$.

Suppose $y^*_a > \widetilde{q}_a$. Take an alternative allocation vector y' given by $y'_a = y^*_a - 1$, $y'_b = y^*_b + 1$, and $y'_i = y^*_i$ for $i \neq a, b$. It is easy to see that $y' \in Y(w)$. The profit for the clearing house by allocating y' equals

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y^*_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_a) > \sum_{i \in I} y^*_i \widetilde{p}_i.$$

The last term is the profit of the clearing house if y^* is allocated. Hence $y^* \notin \Pi(w)$, so $y^* \notin F(w)$. Therefore, if $y^*_b < 0$ and $y^* \in F(w)$, we must have $y^*_a = \widetilde{q}_a$. By the same argument, we must have $y_a = \widetilde{q}_a$ for every allocation $y \in F(w)$ such that $y_b < 0$.

Now suppose there is an allocation $\hat{y} \in F(w)$ such that $\hat{y}_b = 0$ and $\hat{y}_a > \widetilde{q}_a$. According to the result in the first part of the proof, for any seller h that offers $\widetilde{p}_h > \widetilde{p}_a$, $\hat{y}_h = 0$, otherwise $\hat{y} \notin F(w)$. Hence

$$\sum_{\{h \in S : \widetilde{p}_h > \widetilde{p}_a\}} \hat{y}_h \;=\; 0 \;>\; y^*_b \;\geq\; \sum_{\{h \in S : \widetilde{p}_h > \widetilde{p}_a\}} y^*_h.$$

According to the first part of the proof, $y^*_i = \widetilde{q}_i$ for $i \in S$ if $\widetilde{p}_i < \widetilde{p}_b$. Since $\widetilde{p}_b > \widetilde{p}_a$, we have

$$\sum_{\{h \in S : \widetilde{p}_h \leq \widetilde{p}_a\}} \hat{y}_h \;\geq\; \sum_{\{h \in S : \widetilde{p}_h \leq \widetilde{p}_a\}} \widetilde{q}_h \;=\; \sum_{\{h \in S : \widetilde{p}_h \leq \widetilde{p}_a\}} y^*_h.$$

Therefore,

$$\sum_{i \in B} y^*_i = -\sum_{i \in S} y^*_i > -\sum_{i \in S} \hat{y}_i = \sum_{i \in B} \hat{y}_i.$$

Since $\sum_{i \in B} \hat{y}_i < \sum_{i \in B} y^*_i$, there exists at least one buyer, say buyer e, such that $0 \leq \hat{y}_e < y^*_e \leq \widetilde{q}_e$. Since $y^*_e > 0$ and $y^*_a < 0$, from proposition 1 we have $\widetilde{p}_e \geq \widetilde{p}_a$. Therefore $\widetilde{p}_e \geq p_a$, $\hat{y}_a > \widetilde{q}_a$, and $\hat{y}_e < \widetilde{q}_e$, violating proposition 2.

$\square$

Proposition 4. *Given an offer profile w, if buyer $b \in AB(w)$ and seller $s \in AS(w)$, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

Proof. Since $b \in AB(w)$ and $s \in AS(w)$, there must be some $y, y' \in F(w)$ such that $y_b > 0$ and $y'_s < 0$. If $y = y'$, the desired result follows from Proposition 1. Suppose $y \neq y'$. Since $y, y' \in \Pi(w)$, we have

$$\sum_{i \in AB(w)} y_i \widetilde{p}_i + \sum_{i \in AS(w)} y_i \widetilde{p}_i = \sum_{i \in AB(w)} y'_i \widetilde{p}_i + \sum_{i \in AS(w)} y'_i \widetilde{p}_i.$$

Suppose $\widetilde{p}_b < \widetilde{p}_s$. From Proposition 1, we have $y_s = 0$ and $y'_b = 0$. From proposition 3, then, there is no active seller submitting a price higher than p_s, and no active buyer submitting a price lower than p_b. Denote by $\overline{AB}$ the set of active buyers that offer p_b, and $\underline{AS}$ the set of active sellers that offer p_s. From proposition 3, for $i \in AB(w) \backslash \overline{AB}$ and $i \in AS(w) \backslash \underline{AS}$,

$$y'_i = y_i = \widetilde{q}_i.$$

Thus,

$$\sum_{i\in\overline{AB}} y_i p_b + \sum_{i\in\underline{AS}} y_i p_s = \sum_{i\in\overline{AB}} y_i' p_b + \sum_{i\in\underline{AS}} y_i' p_s,$$

which is equivalent to

$$p_b \cdot \left(\sum_{i\in\overline{AB}} y_i - \sum_{i\in\overline{AB}} y_i'\right) = p_s \cdot \left(\sum_{i\in\underline{AS}} y_i' - \sum_{i\in\underline{AS}} y_i\right).$$

Given $\widetilde{p}_b < \widetilde{p}_s$, the equation above implies either

$$\sum_{i\in\overline{AB}} y_i - \sum_{i\in\overline{AB}} y_i' = \sum_{i\in\underline{AS}} y_i' - \sum_{i\in\underline{AS}} y_i = 0$$

or

$$\sum_{i\in\overline{AB}} y_i - \sum_{i\in\overline{AB}} y_i' > \sum_{i\in\underline{AS}} y_i' - \sum_{i\in\underline{AS}} y_i.$$

Since $y_b > 0$, in the first case there must be some buyer c such that $y_c' > 0$ and $\widetilde{p}_c = \widetilde{p}_b < \widetilde{p}_s$. But since $y_s' < 0$, proposition 1 implies $\widetilde{p}_c \geq \widetilde{p}_s$, a contradiction.

In the second case we have

$$\sum_{i\in\overline{AB}} y_i + \sum_{i\in\underline{AS}} y_i > \sum_{i\in\underline{AS}} y_i' + \sum_{i\in\overline{AB}} y_i',$$

which implies

$$\sum_{i\in I} y_i > \sum_{i\in I} y_i'.$$

But since $y, y' \in F(w) \subseteq Y(w)$, we have $\sum_{i\in I} y_i = \sum_{i\in I} y_i' = 0$, a contradiction.

□

Proposition 5. *If $y \in F(w)$, then either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$ or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$. Furthermore, if there is $y^* \in F(w)$ such that $y_i^* = \widetilde{q}_i$ for all $i \in AS(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AS(w)$. Symmetrically, if there is $y^* \in F(w)$ such that $y_i^* = \widetilde{q}_i$ for all $i \in AB(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AB(w)$.*

Proof. For the first part, from proposition 4, if buyer b and seller s are active, we have $p_b \geq p_s$. Thus, from proposition 2, there is no $y \in F(w)$ such that $y_s > \widetilde{q}_s$ and $y_b < \widetilde{q}_b$. Therefore, for every $y \in F(w)$, either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$, or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$.

For the second part, we show the proof for the active sellers' case, since the buyers' case is symmetric. Suppose there is $y^* \in F(w)$ such that $y_i^* = \widetilde{q}_i$ for all $i \in AS(w)$, and $y' \in F(w)$ such that $y_a' > \widetilde{q}_a$ for some $a \in AS(w)$. Then

$$\sum_{i\in B} y_i^* = -\sum_{i\in S} y_i^* > -\sum_{i\in S} y_i' = \sum_{i\in B} y_i'.$$

Therefore there must be an active buyer, say b, such that $y_b' < y_b^* \leq \widetilde{q}_b$. From proposition 4, we have $\widetilde{p}_b \geq \widetilde{p}_a$. But, from proposition 2, $y_a' > \widetilde{q}_q$ and $y_b' < \widetilde{q}_b$ imply $\widetilde{p}_b < \widetilde{p}_a$, a contradiction. □

Proposition 6. *If seller a offers $(p, \widetilde{q}_a)$ in offer profile w and $a \in AS(w)$, then seller b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b < 0$ is also an active seller. Symmetrically, if buyer a offers $(p, \widetilde{q}_a)$ in strategy profile w and $a \in AB(w)$, then buyer b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b > 0$ is also an active buyer.*

Proof. We show the proof for the sellers' case, since the buyers' case is symmetric. Suppose $y_b = 0$ for all $F(w)$. Provided seller a is an active seller, there exists $y \in F(w)$ such that $y_a > 0$. Consider an allocation vector y' that $y'_i = y_i$ for $i \neq a, b$ and $y'_a = y_a - 1$, $y'_b = 1$. It's easy to see that $y' \in Y(w)$. The profit for the clearing house if y' is allocated is equal to

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + p - p = \sum_{i \in I} y_i \widetilde{p}_i,$$

so that $y' \in \Pi(w)$. Thus, either $y' \in F(w)$ or there is $y'' \in F(w)$ such that $y'' \neq y'$ and $|y''_i| \geq |y'_i|$ for all i, so that in either case b is an active trader, contradicting the assumption. Therefore, as long as seller a is an active seller, so is seller b.

□

Proof of Theorem 1

Suppose (p, q) is a competitive equilibrium. First we claim that the offer profile $w = ((p, q_i))$ induces the same outcome with probability one. To see this, since $\widetilde{p}_i = p$ for all i, the arbitrage profit for the clearing house is 0 for each $y \in Y(w)$, so that $\Pi(w) = Y(w)$. Clearly $q \in Y(w)$ since by definition of a competitive equilibrium $q_i \in Q_i$ and $\sum_i q_i = 0$. Moreover, by definition of $Y(w)$, for every $y \in Y(w)$ we have $|y_i| \leq |q_i|$. Hence q ray-dominates any other allocation in $\Pi(w)$ and is the unique element of $F(w)$. Thus, w induces the outcome $(q, (-pq_i))$ with probability one. This is precisely the outcome induced by the competitive equilibrium.

Next, we show that no individual i has an incentive to deviate from the offer profile $w = ((p, q_i))$. We consider deviations for buyers, since the proof for sellers is symmetric. We classify possible individual deviations for $i \in B$ from w into three categories, and show that none of them is profitable.

(i) Consider $w'_i = (p, q'_i)$ such that $Q_i \ni q'_i \neq q_i$. In any outcome with positive probability after that deviation, the utility for i is $u_i(y, -py)$ for some $y \in Q_i$. Since $q_i \in \arg\max_{q \in Q_i} u_i(q, -pq)$, we have that the expected utility after the deviation cannot be larger.

(ii) Consider $w'_i = (p'_i, q'_i)$ such that $q'_i \in Q_i$ and $p'_i < p$. Since every seller $s \in S$ is asking $\widetilde{p}_s = p$, by proposition 1 we must have that in any outcome with positive probability after that deviation $y_i = 0$. But then the expected utility after the deviation is 0, and since $q_i \in \arg\max_{q \in Q_i} u_i(q, -pq)$, we have $u_i(q_i, -pq_i) \geq u_i(0, -p \times 0) = 0$.

(iii) Consider $w'_i = (p'_i, q'_i)$ such that $q'_i \in Q_i$ and $p'_i > p$. Denote by w' the new offer profile. For any $y' \in F(w')$, buyer i gets a payoff of $u_i(y'_i, -p'_i y'_i)$. Note that $u_i(y'_i, -p'_i y'_i) < u_i(y'_i, -py'_i) \leq u_i(q_i, -pq_i)$, where the first inequality follows from $p'_i > p$ and the fact that $u_i(q, -pq)$ is decreasing in p, and the second from $q_i \in \arg\max_{q \in Q_i} u_i(q, -pq)$. It follows that $Eu_i(w') < Eu_i(w)$.

Proof of Theorem 2

The proof comes in several steps. Note that the 'thick market' condition (at least two active traders in the Nash equilibrium) is invoked only in the last step.

Lemma 1. *In each Nash equilibrium, all active sellers offer the same price, and all active buyers offer the same price.*

Proof. We prove the result for the sellers; the proof for the buyers is analogous. Take any offer profile w^* such that two active sellers offer different prices $\overline{p}$ and $\underline{p}$, with $\overline{p} > \underline{p}$. We can claim that the seller, say trader l, who offers $(\underline{p}, \widetilde{q}_l)$ would be better off submitting $(p', \widetilde{q}_l)$ such that $\underline{p} < p' < \overline{p}$.

To see this, from proposition 3, since there is another active seller offering the price $\overline{p}$, seller l sells $|\widetilde{q}_l|$ units when she offers $w_l^* = (\underline{p}, \widetilde{q}_l)$. We can show that seller l sells $|\widetilde{q}_l|$ units as well when she offers $w' = (p', \widetilde{q}_l)$. Suppose there is $y' \in F(w')$ such that $y'_l > \widetilde{q}_l$. As in the last step of the proof of proposition 3, for any $y \in F(w^*)$ we have

$$\sum_{i \in B} y_i = -\sum_{i \in S} y_i > -\sum_{i \in S} y'_i = \sum_{i \in B} y'_i.$$

Therefore there must be an active buyer at w^*, say b, such that $y'_b < y_b \leq \widetilde{q}_b$. From proposition 4, we have $\widetilde{p}_b \geq \overline{p}$. But, from proposition 2, at profile w' we have $y'_l > \widetilde{q}_l$ and $y'_b < \widetilde{q}_b$ implying $\widetilde{p}_b < p' < \overline{p}$, a contradiction.

Thus, by offering $(p', \widetilde{q}_l)$, seller l gets

$$u_l(\widetilde{q}_l, -p'\widetilde{q}_l) = -p'\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj} > -\underline{p}\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj},$$

where the last term is the payoff seller l gets by offering $(\underline{p}, \widetilde{q}_l)$. Hence the seller gets better off by offering $(p', \widetilde{q}_l)$, so that w^* cannot be a Nash equilibrium. □

Lemma 2. *In each Nash equilibrium, all active traders offer the same price.*

Proof. Consider an offer profile w^* such that there is trade and such that all active buyers offer the same price, say p_b, and all active sellers offer the same price, say p_s. From lemma 1, we know that only such profiles, or some profiles such that there is no trade, can be Nash equilibria. By proposition 4, we have $p_s \leq p_b$. We will show that if $p_s < p_b$, at least one active trader has an incentive to deviate, so that w^* cannot be a Nash equilibrium.

If $p_s < p_b$, following proposition 5, we have that either $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$, or $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AB(w^*)$. Suppose $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$ (the argument for the other case is analogous). Note that if there are inactive sellers in w^*, for any such seller h we have $\widetilde{p}_h > p_s$ or $\widetilde{q}_h = 0$. If $\widetilde{p}_h < p_s$, then following proposition 3 we have $y_h = \widetilde{q}_h$ for all $y \in F(w^*)$, so the seller can be inactive only if $\widetilde{q}_h = 0$. If $\widetilde{p}_h = p_s$, according to proposition 6 there must be some $y \in F(w^*)$ that $y_h < 0$ unless $\widetilde{q}_h = 0$. Denote by $p_{\neg s}$ the lowest price offered with a non-zero quantity by inactive sellers, if there is any, and note that in that case $p_{\neg s} > p_s$.

We claim that if $p_s < p_b$, an active seller, say a, would have an incentive to deviate from $w_a^* = (p_s, \widetilde{q}_a)$ to $w'_a = (p', \widetilde{q}_a)$, where $p' \in (p_s, \min\{p_b, p_{\neg s}\})$ if there are non-zero quantity inactive sellers, and $p' \in (p_s, p_b)$ otherwise.

To prove the claim, we argue first that for any $y' \in F(w')$ we have $y'_a = \widetilde{q}_a$. Suppose there is a $y' \in F(w')$ such that $y'_a > \widetilde{q}_a$. Then, from proposition 3, inactive sellers at w^* remain so at w' since $p_{\neg s} > p'$. Therefore $\sum_{i \in S} y'_i > \sum_{i \in S} y_i$ for every $y \in F(w^*)$. Thus, for any $y \in F(w^*)$,

$$\sum_{i \in B} y'_i = -\sum_{i \in S} y'_i < -\sum_{i \in S} y_i = \sum_{i \in B} y_i.$$

Therefore there must be an active buyer at w^*, say h, who offers p_b and gets $y'_h < y_h \leq \widetilde{q}_h$. But, from proposition 2, at profile w' we have $y'_a > \widetilde{q}_a$ and $y'_h < \widetilde{q}_h$ implying $p_b < p'$, a contradiction.

From the previous argument, by offering w' instead of w^*, seller a is allocated $\widetilde{q}_a$, and gets a utility of

$$u_a(\widetilde{q}_a, -p'\widetilde{q}_a) = -p'\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} > -p_s\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} = u_a(\widetilde{q}_a, -p_s\widetilde{q}_a).$$

Hence seller a gets better off by offering w', so that w^* cannot be a Nash equilibrium. □

Lemma 3. *In each Nash equilibrium, every trader is indifferent between all outcomes that occur with positive probability.*

Proof. Consider an offer profile w^* such that all active traders, if there is any, offer the same price, say p. From lemma 2, we know that only such profiles can be Nash equilibria if trades happen with positive probability. Take trader a, a seller, for example. If seller a is inactive, then her utility is 0 for any positive probability outcome. Suppose a is active, and moreover there are $y, y'' \in F(w^*)$ that $u_a(y_a, -py_a) > u_i(y''_a, -py''_a)$. We can show that in this case, w cannot be a Nash equilibrium.

Since $F(w^*)$ is finite, there is some $y^* \in F(w^*)$ such that $u_a(y^*_a, -py^*_a) \geq u_a(y_a, -py_a)$ for all $y \in F(w^*)$ and moreover $u_a(y^*_a, -py^*_a) > u_a(y''_a, -py''_a)$. Since y'' has positive probability, $u_a(y^*_a, -py^*_a) > Eu_a(w)$. By continuity, there is some $p' < p$ such that $u_a(y^*_a, -py^*_a) > u_a(y^*_a, -p'y^*_a) > Eu_a(w)$.

We claim that if seller a offers $w'_a = (y^*_a, p')$, then $y'_a = y^*_a$ for every $y' \in F(w')$, so that the seller obtains $u_a(y^*_a, -p'y^*_a)$ which is a profitable deviation from w^* by the inequality above. The claim implies that w^* cannot be a Nash equilibrium. To verify the claim, suppose first that there is another seller h that is active at w'; since seller h offers the price $p > p'$, the claim follows from proposition 3. Suppose that no other seller is active at w', then if $y'_a > y^*_a$ we get for any $y' \in F(w')$,

$$\sum_{i \in B} y'_i = -\sum_{i \in S} y'_i < -\sum_{i \in S} y^*_i = \sum_{i \in B} y^*_i.$$

Then there must be some buyer, say b, such that $y'_b < y^*_b \leq \widetilde{q}_b$. Since there is also a seller, seller a, such that $y'_a > y^*_a$ and moreover this seller offers a price p' below the price offered by the buyer, we get a contradiction with proposition 2. □

Lemma 4. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all non-active traders are allocated utility-maximizing quantities.*

Proof. In the proof of this and the following lemmas, let w^* be a Nash equilibrium with active trading, and (invoking lemma 2) let p^* be the price offered by all active traders. We focus on sellers; the proof for the buyers is analogous.

As shown in the second paragraph of lemma 2, non-active sellers offer $\widetilde{p}_i > p^*$ or $\widetilde{q}_i = 0$. Therefore, they get $y_i = 0$ for all $y \in F(w^*)$, and thus obtain $Eu_i(w^*) = 0$. We claim that for inactive sellers, $y_i = 0$ is utility-maximizing given price p^*. Equivalently, we claim that $r_{i1} \geq p^*$.

To see this, suppose trader i is an inactive seller and $r_{i1} < p^*$. Consider a deviation for i to $w'_i = (p^*, -1)$. By proposition 6, if seller i is inactive under the offer profile w', so is every seller in $AS(w^*)$ under the offer profile w', and by proposition 3 so is every seller. But this would violate proposition 2, since there are trades in each side of the market active under w^* and thus offering p^* should induce positive probability to trade. Hence, there exists $y' \in F(w')$ such that $y'_i = -1$. Since $u_i(y'_i, -p^* y'_i) = p^* - r_{i1} > 0$, by deviating to offer $(p^*, -1)$, agent i would have $Eu_i(w') > 0$, so that w^* would not be a Nash equilibrium.

□

Lemma 5. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all active traders are allocated quantities that are either utility-maximizing or involve less in absolute value than the utility-maximizing trade.*

Proof. For a given active seller, say s, let $\underline{\delta}_s$ and $\overline{\delta}_s$ be the minimal and the maximal element, respectively, of the set $\arg\max_{q_s \in Q_s} u_s(q_s, -p^* q_s)$, so that $-k \leq \underline{\delta}_s \leq \overline{\delta}_s \leq 0$. From the utility maximization problem, it follows that every $x \in Q_s$ such that $\underline{\delta}_s \leq x \leq \overline{\delta}_s$ is also a utility maximizer.

We claim that for every $y \in F(w^*)$ we have $y_s \geq \underline{\delta}_s$ so that either the seller is allocated an optimal trade or a smaller (in absolute value) than optimal trade. For suppose there is $y \in F(w^*)$ such that $y_s < \underline{\delta}_s$ so that $u_s(y_s, -p^* y) < u_s(\underline{\delta}_s, -p^* \underline{\delta}_s)$. If $\underline{\delta}_s = 0$ or $p^* = 0$, it follows that $u_s(y_s, -p^* y) < 0$, and by lemma 3, $Eu_s(w^*) < 0$. But then trader s can deviate to $(p^*, 0)$ and guarantee an expected utility of zero, so that w^* cannot be a Nash equilibrium. Suppose instead that $\underline{\delta}_s < 0$ and $p^* > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^* y) < u_s(\underline{\delta}_s, -p' \underline{\delta}_s) < u_s(\underline{\delta}_s, -p^* \underline{\delta}_s).$$

Now consider a deviation by s to $w'_s = (p', \underline{\delta}_s)$. We show that such deviation guarantees $y'_s = \underline{\delta}_s$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(\underline{\delta}_s, -p' \underline{\delta}_s) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that $y'_s > \underline{\delta}_s$. Since $p' < p^*$, and all other sellers offer a price equal or larger than p^* or a quantity equal to zero, it follows from proposition 3 that for all other $i \in S$ we have $y'_i = 0$. Therefore

$$\sum_{i \in B} y_i = -\sum_{i \in S} y_i \geq -y_s > -\underline{\delta}_s = -\sum_{i \in S} y'_i = \sum_{i \in B} y'_i.$$

But then there must be a buyer, say a, such that $y'_a < y_a \leq \tilde{q}_a$ offering price $p^* > p'$, contradicting proposition 2.

$\square$

Lemma 6. *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, if there are two or more active traders on the same side of the market, then all traders on this side of the market are allocated utility-maximizing quantities.*

Proof. We claim that if there are at least two active sellers, then every $y \in F(w^*)$ satisfies $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$ and is therefore a utility maximizer.

In lemma 5 we have shown in every positive probability allocation y, active sellers are allocated quantities that are either utility-maximizing given the price or involve less trade ($\underline{\delta}_s \leq y_s \leq 0$) so we need only focus on active sellers.

Now suppose there are two active sellers, say s and h. If $y_s < \underline{\delta}_s$ for any $y \in F(w^*)$, we have that w^* cannot be a Nash equilibrium by the previous step. If $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$, the claim follows from lemma 3. In the last part of this proof, we show that if there is a $y \in F(w^*)$ such that $y_s > \overline{\delta}_s$, w^* cannot be a Nash equilibrium.

Since $|y_s| < |\overline{\delta}_s|$, from the utility maximization problem we must have $r_{|y_s|+1} < p^*$. Hence $u_s(y_s - 1, -p^*(y_s - 1)) - u_s(y_s, -p^* y_s) = p^* - r_{|y_s|+1} > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^* y) < u_s(y_s - 1, -p'(y_s - 1)) < u_s(y_s - 1, -p^*(y_s - 1))).$$

Also, for any $y, y'' \in F(w^*)$, we have $y_s = y''_s$. Suppose there exists $y, y'' \in F(w^*)$ such that $y_s < y''_s$, then

$$u_s(y''_s, -p^* y''_s) - u_s(y_s, -p^* y_s) = -p^*(y''_s - y_s) + \sum_{j=|y''_s|+1}^{|y_s|} r_{sj} < 0,$$

contradicting lemma 3.

Now consider a deviation by s to $w'_s = (p', y_s - 1)$. We show that such deviation guarantees $y'_s = y_s - 1$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(y_s - 1, -p'(y_s - 1)) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that $y'_s > y_s - 1$. Since $p' < p^*$, and all other sellers offer a price equal or larger than p^* or a quantity equal to zero, it follows from proposition 3 that for all other $i \in S$ we have $y'_i = 0$. Therefore, take any $y'' \in F(w^*)$ such that $y''_h < 0$,

$$\sum_{i \in B} y''_i = -\sum_{i \in S} y''_i \geq -y_s - y''_h \geq -y_s + 1 > -y'_s = -\sum_{i \in S} y'_i = \sum_{i \in B} y'_i$$

But then there must be a buyer, say a, such that $y'_a < y''_a \leq \tilde{q}_a$ offering price $p^* > p'$, contradicting proposition 2. $\square$

Since the market clearing condition in the equilibrium definition is satisfied by any allocation induced by any offer profile, theorem 2 follows from lemma 6.

Properties of v_b and v_s

Lemma 7. *In any competitive equilibrium $(p,q) \in \xi(r)$ that contains the smallest number of transactions, the lowest reservation value of buyers' traded unit(s) is equal to v_b, and the highest reservation value of sellers' traded unit(s) is equal to v_s.*

Proof. We show the proof for v_b; the proof for v_s is analogous. By definition of v_b, there is a competitive equilibrium $(\hat{p},\hat{q})$ such that every unit bought has a buyer's valuation greater than or equal to v_b. Suppose there is a competitive equilibrium $(\tilde{p},\tilde{q})$ such that a buyer, say $i \in B$, buys a unit with valuation strictly below v_b. Then it must be the case that $\tilde{p} < v_b$. But then we have that $\tilde{q}_i > \hat{q}_i$ and for every $j \in B \setminus \{i\}$, $\tilde{q}_j \geq \hat{q}_j$, so that strictly more units are traded in $(\tilde{p},\tilde{q})$ than in $(\hat{p},\hat{q})$. $\square$

Proof of Theorem 3

First we prove the condition in the statement of the theorem is sufficient. Suppose w^* is a Nash equilibrium with active trading, and suppose there are at least two inframarginal sellers and at least two weakly inframarginal buyers. (The other case is analogous.) From lemma 1 and lemma 2, all active traders offer the same price, say p^*. Denote $\underline{\delta}_i$ and $\overline{\delta}_i$ the minimal and maximal element, respectively, of the set $\arg\max_{q_i \in Q_i} u_i(q_i, -p^* q_i)$. From lemma 5, for any $y \in F(w^*)$, we have $\underline{\delta}_i \leq y_i \leq 0$ for every active seller i, and $0 \leq y_i \leq \overline{\delta}_i$ for every active buyer i, and moreover from lemma 4, non-active traders acquire utility-maximizing quantities given p^*. That is, no one trades in excess of their utility-maximizing quantity.

Consider first the case $p^* > v_s$. We claim that every inframarginal seller must be active. For suppose an inframarginal seller i is not active; then the seller is making a payoff equal to zero in every allocation $y \in F(w^*)$. But by deviating unilaterally to $w_i'(p,1)$ for any $v_s < p < p^*$, the seller can guarantee herself a positive payoff $u_i(-1,p) = -r_{i1} + p > -v_s + p^* > 0$ in every allocation with positive probability given the new offer profile. Hence, two or more sellers are active in w^*. If two or more buyers are active in w^*, then applying theorem 2, p^* is a competitive price and all the outcomes from the Nash equilibrium are competitive.

If only one buyer is active in w^*, say buyer a, we must have that at least one weakly inframarginal buyer, say buyer c, who is not active in w^*. Since c is not active in w^*, we must have $p^* \geq r_{c1}$; otherwise c has a profitable deviation. Therefore $p^* \geq r_{c1} \geq \underline{p}$. If $p^* > \overline{p}$, then for every $y \in F(w^*)$,

$$\sum_{i \in B} y_i \leq \sum_{i \in B} \overline{\delta}_i < -\sum_{i \in S} \underline{\delta}_i \leq -\sum_{i \in S} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 6 which implies that for all the active sellers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them, and from lemma 5 which implies that for any active buyer $y_i \leq \overline{\delta}_i$. The strict inequality in the middle is a result of the price being higher than any competitive price. Hence $\underline{p} \leq p^* \leq \overline{p}$ so that p^* is a competitive price.

Now suppose that there is an allocation $y \in F(w^*)$ such that $y_a < \underline{\delta}_a$. Since p^* is competitive, in any competitive equilibrium allocation (q_i), we have $-\sum_{i \in S} q_i \geq \underline{\delta}_a$.

Thus in every competitive equilibrium at price p^*, there exists at least one seller s that has $q_s < y_s$. Since $y_s, q_s \in [\underline{\delta}_s, \overline{\delta}_s]$, we have $r_{s,|q_s|} = p^*$. Hence for any competitive equilibrium at p^*, there is at least a traded unit with reservation value p^* for a seller. By definition of v_s this implies $p^* \leq v_s$, a contradiction to the assumption. Therefore for the only active buyer a, $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$. Hence, all traders obtain utility-maximizing quantities given p^*, and every outcome $y \in F(w^*)$ is competitive.

Consider the remaining case $p^* \leq v_s$. Since $p^* < r_{1i}$ for every weakly inframarginal buyer, it follows that there are at least two active buyers in Nash equilibrium and moreover every buyer chooses utility-maximizing quantities given p^*. As in the previous proof, if there are two or more active sellers, then, from theorem 2, all outcomes in $F(w^*)$ are competitive. Similarly, if there is a unique active seller a and $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$, then all traders obtain utility-maximizing quantities given p^*, and every outcome $y \in F(w^*)$ is competitive. The remaining case is that there is a unique active seller a and $\overline{\delta}_a < y_a < 0$, so that $\sum_{i \in B} y_i = -y_s < -\overline{\delta}_s$.

Suppose $p^* = v_s = \underline{p}$. Since p^* is a competitive price, in every competitive equilibrium allocation (q_i), we have $\sum_{i \in B} q_i \geq -\overline{\delta}_s$; i.e. aggregate demand should be able to meet an individual seller's supply. Thus in every competitive equilibrium at p^*, there exists at least one buyer b that has $q_b > y_b$. Since $y_b, q_b \in [\underline{\delta}_b, \overline{\delta}_b]$, we have $r_{b,q_b} = p^*$. Hence in every competitive equilibrium at p^*, there is at least one traded unit with reservation value p^* for a buyer. By definition of v_b, this implies $p^* \geq v_b$. Using $v_b > v_s$ we get a contradiction to the assumption $p^* = v_s$.

Finally, suppose $p^* = v_s < \underline{p}$ or $p^* < v_s$. In either case, $p^* < \underline{p}$, and

$$-\sum_{i \in S} y_i \leq -\sum_{i \in S} \overline{\delta}_i < \sum_{i \in B} \underline{\delta}_i \leq \sum_{i \in B} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 5 which implies that for any active seller $y_i \geq \underline{\delta}_i$ and from lemma 6 which implies that for all the active buyers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them. The strict inequality in the middle is a result of the price being lower than any competitive price.

This finishes the proof of sufficiency. We now prove that the condition is necessary. Since at least two units are traded in every competitive equilibrium, there is at least one inframarginal trader on each side of the market. Possible violations of the condition in the theorem are that, among the remainder of traders, either (a) there is no additional weakly inframarginal trader on one side of the market, or (b) there is no additional inframarginal trader in either side.

Consider case (a), and suppose without loss of generality that trader 1 is the unique weakly inframarginal seller, so that every seller $i \in S \setminus \{1\}$ is such that either $r_{i1} \geq v_b$ or $r_{i1} > \overline{p}$; recall that each of these conditions imply $r_{i1} > v_s$. Take a competitive equilibrium that has the smallest number of units traded, and denote the allocation by $\hat{q} = (\hat{q}_i)$. From lemma 7, $\hat{q}_i = 0$ for every seller $i \in S \setminus \{1\}$. From lemma 7 as well, a unit of value v_b is bought by at least one buyer, say without loss of generality buyer 2, and moreover for every buyer j such that $q_j > 0$ we must have $r_{j,\hat{q}_j} \geq v_b$. Recall that the highest equilibrium price $\overline{p}$ satisfies $\overline{p} \leq v_b$, and moreover $(\overline{p}, \hat{q})$ is a competitive equilibrium.[7] Suppose first that $\overline{p} = v_b$. Consider the offer profile w such that $w_1 = (v_b, \hat{q}_1 + 1)$ (seller 1 sells one fewer unit than in the competitive equilibrium), $w_2 = (v_b, \hat{q}_2 - 1)$ (buyer 2 buys one fewer unit), and $w_i = (v_b, \hat{q}_i)$ for every $i \neq 1,2$. It is easy to check that no trader has a profitable deviation; buyer 2 in particular is indifferent between buying one more unit or not.

Now suppose that $\overline{p} < v_b$. Define

$$\tilde{p} = \begin{cases} \min\left\{\min_{i \in S\setminus\{1\}} r_{i1}, v_b\right\} & \text{if } S\setminus\{1\} \neq \emptyset \\ v_b & \text{if } S\setminus\{1\} = \emptyset \end{cases},$$

and consider the offer profile $\tilde{w}$ such that $\tilde{w}_i = (\tilde{p}, \hat{q}_i)$ for all $i \in S \cup B$. It is easy to check that no trader has a profitable deviation. But the induced outcome is not competitive since $\tilde{p} > \overline{p}$.

Consider case (b), and suppose without loss of generality that trader 1 is the unique inframarginal seller and that trader 2 is the unique inframarginal buyer, so that for every seller $i \in S\setminus\{1\}$ and buyer $j \in B\setminus\{2\}$, $r_{i1} > v_s$ and $r_{j1} < v_b$. Take a competitive equilibrium $(\hat{p}, \hat{q})$ that has the smallest number of units traded. Since $v_s \leq \hat{p} \leq v_b$, traders 1 and 2 are the only traders who are trading in $\hat{q}$. Consider the offer profile $w_1 = (\hat{p}, -1)$, $w_2 = (\hat{p}, 1)$, and $w_k = (\hat{p}, 0)$ for every $k \in S \cup B \setminus \{1,2\}$. No trader has a profitable deviation, but this offer profile induces an allocation which is not competitive under the assumption that at least two units are traded in competitive equilibrium.

Appendix B: Additional graphs

See overleaf.

[7] In quasilinear economies, the set of competitive equilibria is the product of the set of competitive allocations and the set of competitive equilibrium prices.

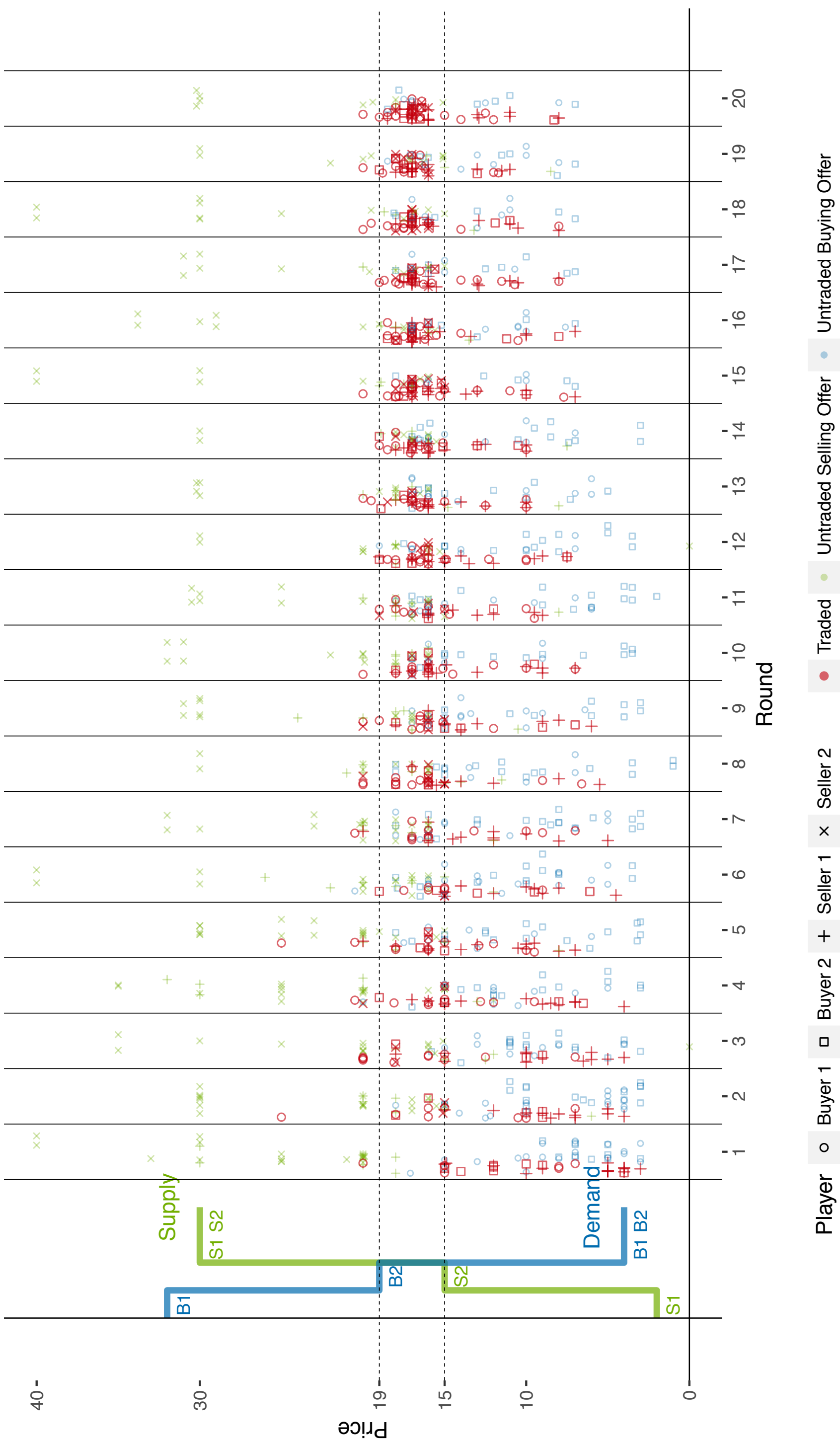


Figure B8: Offers under \$45 and prices in CH Competitive treatment

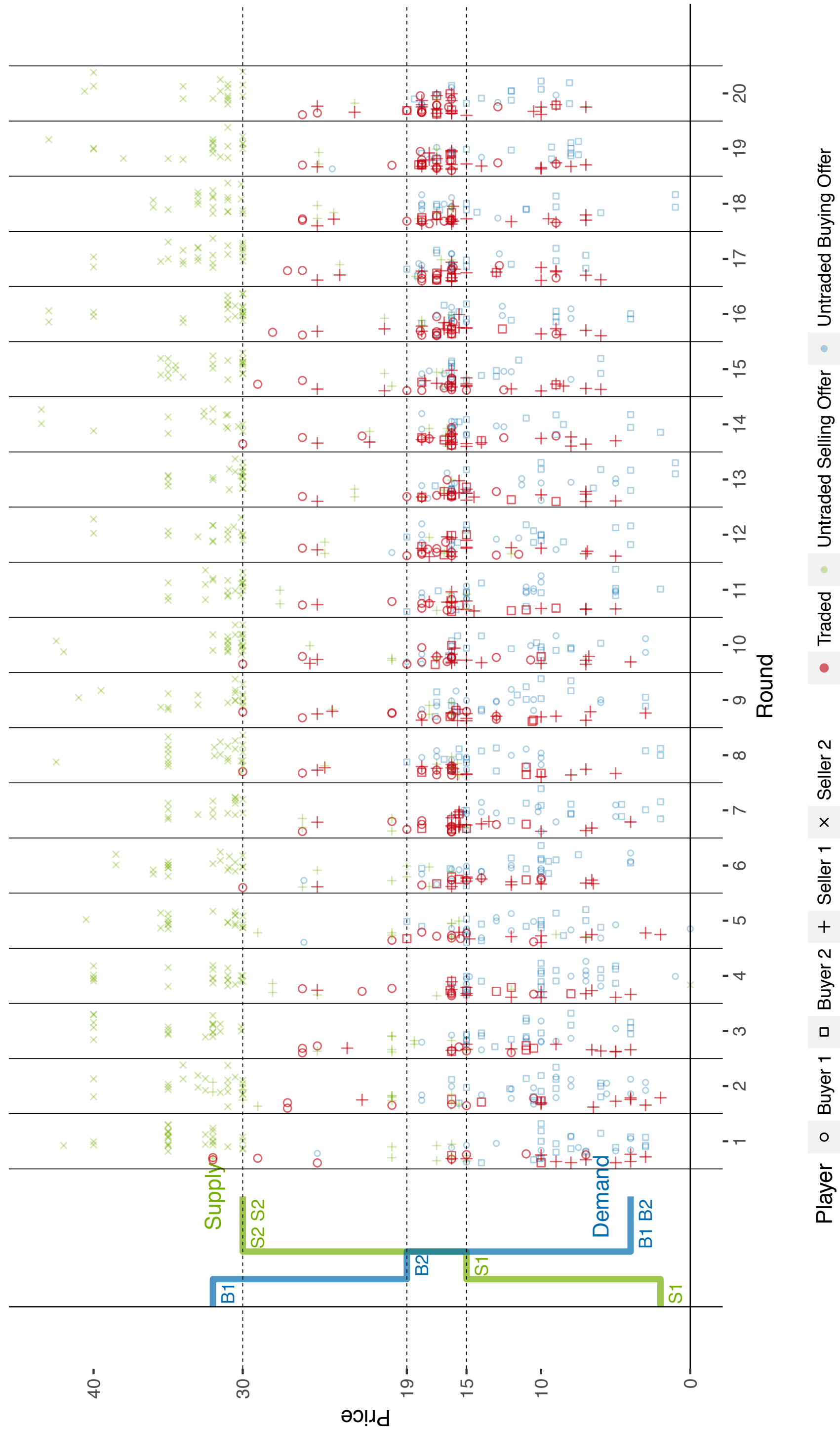


Figure B9: Offers under $45 and prices in CH Monopoly treatment

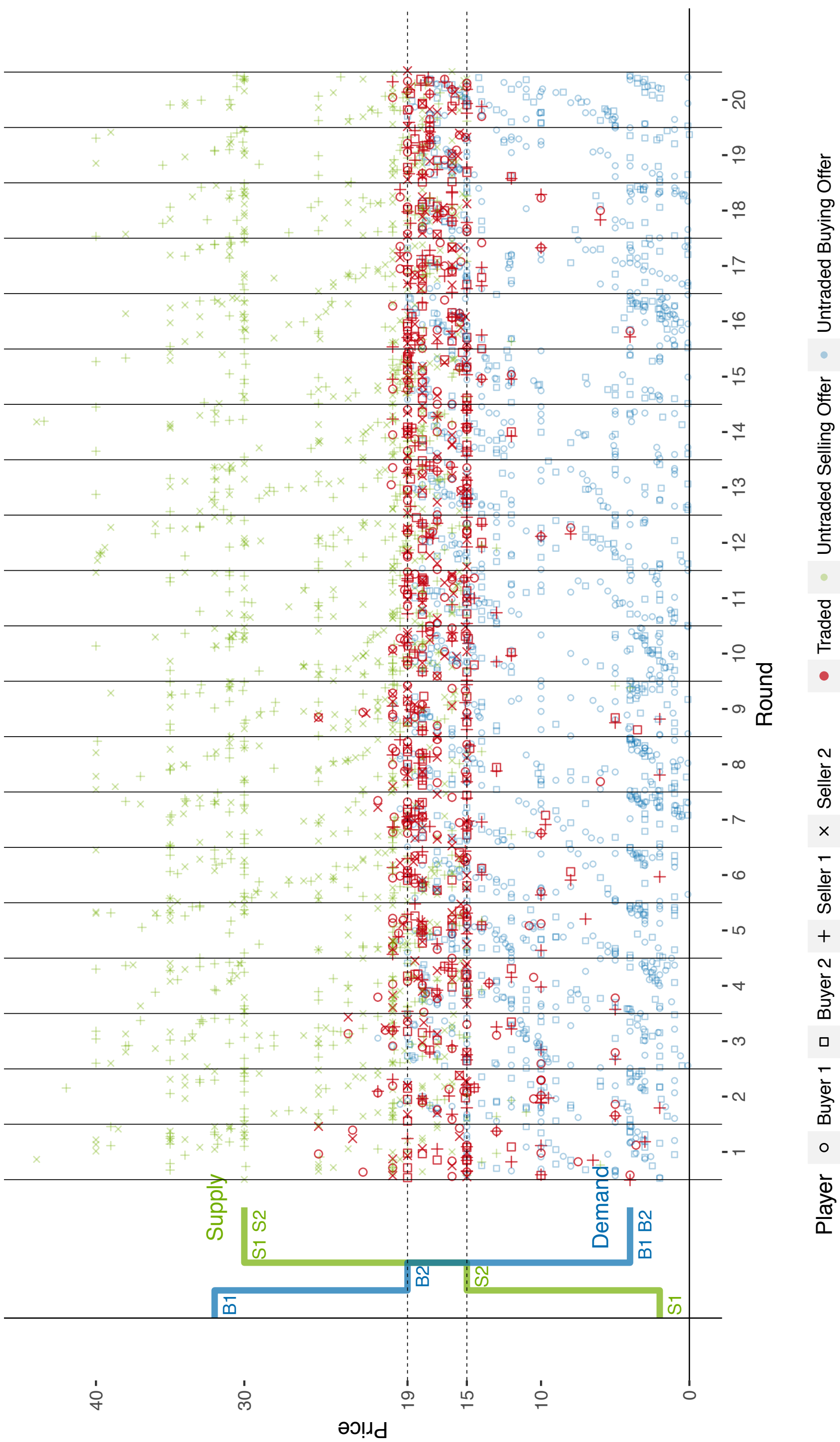


Figure B10: Offers under $45 and prices in DA Competitive treatment

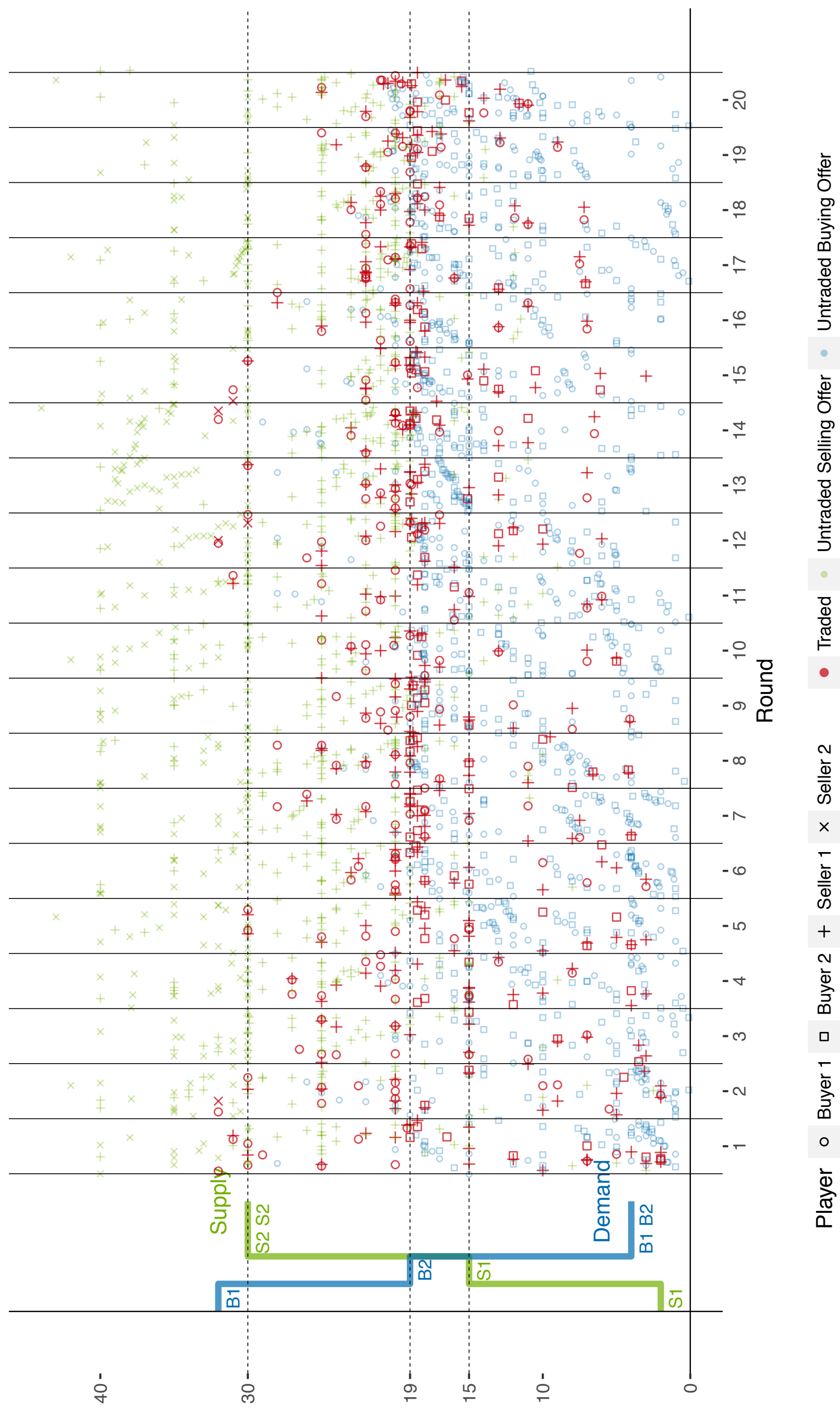


Figure B11: Offers under $45 and prices in DA Monopoly treatment

Appendix C: Instructions and Quizzes

Instructions for CH treatments

Instructions

Welcome to today's experiment! You have earned $5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. Your group and your role will remain the same throughout the experiment. Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

Payoffs

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of $4 from buying the first unit, and $3 from buying the second. If Buyer A obtains 2 units at the unit price of $2, then

$$\text{Buyer A's payoff} = \underbrace{(\$4+\$3)}_{\text{Values}} - \underbrace{(\$2+\$2)}_{\text{Payment}} = \$3$$

Suppose Seller A sells 1 unit at the price of $5.6, and her cost is $1 for selling the first unit and $3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

Since Seller A does not sell the second unit, only the cost of the first unit incurs.

If a participant does not trade in a round, her payoff from that round is $0.

The payoffs from different rounds do not accrue. At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the $5 show-up bonus.

How to trade

Each group trades in its own market. In each round, the market opens for 2 minutes, during which each participant can submit an offer. In a buying offer, a buyer submits a unit price, together with how many units (1 or 2) she would like to buy for that price. In a selling offer, a seller submits a unit price, and how many units (1 or 2) she would like to sell for that price. The offer you submit will NOT be shown to any other participant.

Please note that you can submit only ONE offer in each round, and you cannot revise your offer once you submit it.

After two minutes, or once every participant has submitted a unit price and quantity, transactions will be determined under the rules below, as demonstrated in the following example.

Example

Suppose the submitted offers are as follows.
Buyer A: buying offer for 1 unit, at the unit price of $3
Buyer B: buying offer for 2 units, at the unit price of $1
Seller A: selling offer for 1 unit, at the unit price of $4
Seller B: selling offer for 1 unit, at the unit price of $2.

Please note that this example is only for demonstration of the procedure, the submitted offers will NOT be shown to any participant in the experiment.

- **Sort Orders** Firstly, buying offers and selling offers will be sorted separately. If an offer contains two units (eg. Buyer B's offer), it will be split into TWO IDENTICAL offers, each containing one unit. Buying offers for each unit will be queued in descending order, and selling offers for each unit will be queued in ascending order, as the following table shows.

Buying offers for one unit (high to low)	Selling offers for one unit (low to high)
$3 (from Buyer A)	$2 (from Seller B)
$1 (from Buyer B)	$4 (from Seller A)
$1 (from Buyer B)	

  In case of tied buying offers or tied selling offers, the order of them will be randomly determined.

- **Trade Units** After the orders are sorted, each buying offer and selling offer at the same position in the queues will be compared. As long as the buying price is no lower than the selling price, the corresponding buyer and seller make a trade.

The first buying offer in the queue ($3 from Buyer A) and the first selling offer ($2 from Seller B) make a trade since 3>2. The second buying offer and selling offer cannot trade since the buying price ($1 from Buyer B) is lower than the selling price ($4 from Seller A). The third buying offer cannot be fulfilled since there is not a selling offer corresponding to it. By this procedure, the buying offer with higher price is more likely to be fulfilled, and so is the selling offer with lower price.

- **Prices** When a trade happens, the buyer will pay the price she offered and get one unit of the good, and the seller will receive the price she asked for and sell one unit of the good. In this example, one unit of the good is traded. Buyer A pays $3 for the unit she bought, as she offered to. Seller B gets $2 for the unit she sells, as she asked for.

In each round, a participant who does not submit any offer will not make any trade. To prevent losing money, a buyer/seller cannot submit an offer that could cause a loss for her.

Summary of Each Round

The market for each group opens at the beginning of each round. After each participant in your group submits an offer or when the market closes, you will be informed of how many units you trade, and your payoff in the current round. Also, the price(s) for each traded unit in your market will be revealed anonymously to all participants in your group. You will NOT be informed of the buying/selling offers that do not result in trade.

This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be a practice round, which does not count towards payment.
Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.

Quiz for CH treatments

Quiz

1. True or False. Circle your answers.

Your role (buyer or seller) will remain the same in all of the rounds.	T	F
Your group does not change throughout the experiment.	T	F
In each round, you can revise your offer after you submit it.	T	F
Your costs or values will not change between rounds.	T	F
Your offer in each round will not be shown to other participants.	T	F

2. Suppose the offers submitted are as follows.

Buyer A: buying offer for 2 units, at the unit price of \$3
Buyer B: buying offer for 1 unit, at the unit price of \$5
Seller A: selling offer for 2 units, at the unit price of \$1
Seller B: selling offer for 1 unit, at the unit price of \$2.

(a) Use the procedure demonstrated in the instructions, fill out the buying and selling offers in the table.

Buying offers for one unit (high to low)	Selling offers for one unit (low to high)
\$5 (from Buyer B)	\$1 (from Seller A)
\$_____ (from Buyer _____)	\$1 (from Seller A)
\$3 (from Buyer A)	\$_____ (from Seller _____)

(b) How many units does Buyer A buy? ______ unit(s)

(c) How much does Buyer A pay for the unit(s) she buys <u>in total</u> ? \$______

(d) Suppose the first unit Buyer A obtains will generate a value of \$5 to her, and the second unit she obtains will generate \$4. What is Buyer A's payoff here?

$$\text{Buyer A's payoff} = \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Value(s)}} - \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Payment}} = \$\_\_\_\_\_\_\_\_\_\_$$

(e) Suppose the first unit Seller B sells will cost her \$0.5, and the second unit she sells will cost \$2.5. What is Seller B's payoff here?

$$\text{Seller B's payoff} = \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Revenue}} - \underbrace{\$\_\_\_\_\_\_\_\_\_\_}_{\text{Cost(s)}} = \$\_\_\_\_\_\_\_\_\_\_$$

Instructions for DA treatments

Instructions

Welcome to today's experiment! You have earned \$5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. <u>Your group and your role will remain the same throughout the experiment.</u> Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

Payoffs

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of \$4 from buying the first unit, and \$3 from buying the second. If Buyer A obtains the first unit at the price of \$2 and the second unit at the price of \$1, then

$$\text{Buyer A's payoff} = \underbrace{(\$4+\$3)}_{\text{Values}} - \underbrace{(\$2+\$1)}_{\text{Payment}} = \$4$$

Suppose Seller A sells 1 unit at the price of \$5.6, and her cost is \$1 for selling the first unit and \$3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

<u>Since Seller A does not sell the second unit, only the cost of the first unit incurs.</u>

If a participant does not trade in a round, her payoff from that round is \$0.

<u>The payoffs from different rounds do not accrue.</u> At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the \$5 show-up bonus.

How to trade

Each group trades in its own market. In each round, the market opens for a maximum of two minutes, during which each participant can submit offers. In a buying offer, a buyer submits a price she is willing to buy a unit at. In a selling offer, a seller submits a price she is willing to sell a unit at. <u>For each participant, only after her first unit is traded can she trade her second unit.</u>

The timer on the screen counts down the time remaining for the current round. The timer starts from two minutes at the beginning of each round, then jumps to 20 seconds once a participant attempts to submit an offer, and restarts from 20 seconds every time a participant attempts to submit an offer. The round finishes if two minutes elapse, or if no new attempt occurs within 20 seconds of the last attempt, whichever occurs first.

The attached pages are screen shots of the interface for a seller and a buyer in the same market. Screen shot 1 is for the seller. Screen shot 2 is for the buyer.

From left to right in the upper part of the interface are the ***Submit Your Offer*** section, where you can enter the price for each of your offers; the section for ***general information***, where you can see the number of rounds, your role, time remaining in the current round, and your real-time payoff in the current round; ***Your Values/Costs*** section, where you can see the values or costs for your units and whether they are traded or not.

On the lower part of the interface, from left to right are the ***Selling Offers*** section, which lists the selling offers from low to high; the ***Buying Offers*** section, which lists the buying offers from high to low; the ***Transactions*** section, which displays all transactions in your market in the current round. Your own offers and transactions will be highlighted on the lists.

- **How to Sell**
  - **Offer to Sell**

    You can offer to sell one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be lower than the lowest selling offer at the time, which is the top one on the ***Selling Offers*** list. If you make a new offer, it will replace your previous offer.

    As shown in the screen shots, the lowest selling offer is \$3, so if any of the sellers wants to make a new offer, she has to offer a price lower than \$3.

    To prevent losing money, you cannot submit an offer that could cause a loss for you.
  - **Accept A Buying Offer**

    You can sell one unit by submitting a price equal to the highest buying offer, which is the top one on the ***Buying Offers*** list. By doing so, you sell the unit to the buyer and incur the cost, the buyer pays you the price you submitted. (If you submit a price lower than the highest buying offer, you sell the unit at the price you submit.) In the example from the screen shots, the highest buying offer is \$2, if a seller submits an offer of \$2, she sells the unit to the buyer, and the buyer pays her \$2.
  - **Transactions**

    There are two ways you sell one unit. Your selling offer is accepted by a buyer, or you accept a buying offer. When you sell one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to sell your second unit or accept another buying offer on the list. The rules are the same as for the first unit.
- **How to Buy**

– **Offer to Buy**

  You can offer to buy one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be higher than the highest buying offer at the time, which is the top one on the ***Buying Offers*** list. If you make a new offer, it will replace your previous offer.

  As shown in the screen shots, the highest buying offer is $2, so if any of the buyers wants to make a new offer, she has to offer a price higher than $2.

  To prevent losing money, you cannot submit an offer that could cause a loss for you.

– **Accept A Selling Offer**

  You can buy one unit by submitting a price equal to the lowest selling offer, which is the top one on the ***Selling Offers*** list. By doing so, you buy the unit from the seller and obtain the value, and pay the seller the price you submitted. (If you submit a price higher than the lowest selling offer, you buy the unit at the price you submit.) In the example from the screen shots, the lowest selling offer is $3, if a buyer submits an offer of $3, she buys the unit from the seller, and pays the seller $3.

– **Transactions**

  There are two ways you buy one unit. Your buying offer is accepted by a seller, or you accept a selling offer. When you buy one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to buy your second unit or accept another selling offer on the list. The rules are the same as for the first unit.

Summary of Each Round

The market for each group opens at the beginning of each round. A seller can make selling offers, or accept buying offers, by submitting prices on the interface. A buyer can make buying offers, or accept selling offers, by submitting prices on the interface. When an offer is accepted, a transaction happens. Offers, transactions and your payoff in the current round will be displayed on your screen.

This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be one practice round, which does not count towards payment.

Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.

Quiz for DA treatments

Quiz

1. True or False. Circle your answers.

2. Suppose you are a **buyer**, and the lists of offers are as follows. Your offer is highlighted.

Your role (buyer or seller) will remain the same in all of the rounds.	T	F
Your group does not change throughout the experiment.	T	F
Your costs or values will not change between rounds.	T	F
You can submit offers for both of your units at the same time.	T	F

Selling Offers	Buying Offers
\$4	\$3
\$5	\$1

(a) Which of the following prices can you submit as a new offer? Circle your answer.

A. 2 B. 0.5 C. 3.7 D. 1.5

(b) Which of the following prices can you submit to accept the selling offer of \$4? Circle you answer.

A. 4 B. 2.5 C. 1.2 D. 3

(c) If you accept the lowest selling offer on the list, and your values for the first and second unit are \$7 and \$6 respectively, what is your payoff?

$$\text{Your payoff} = \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Value(s)}} - \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Payment}} = \$\_\_\_\_\_\_\_\_$$

3. Suppose you are a **seller**, and the lists of offers are as follows. Your offer is highlighted.

Selling Offers	Buying Offers
\$4	\$3
\$5	\$1

(a) Which of the following prices can you submit as a new offer? Circle your answer.

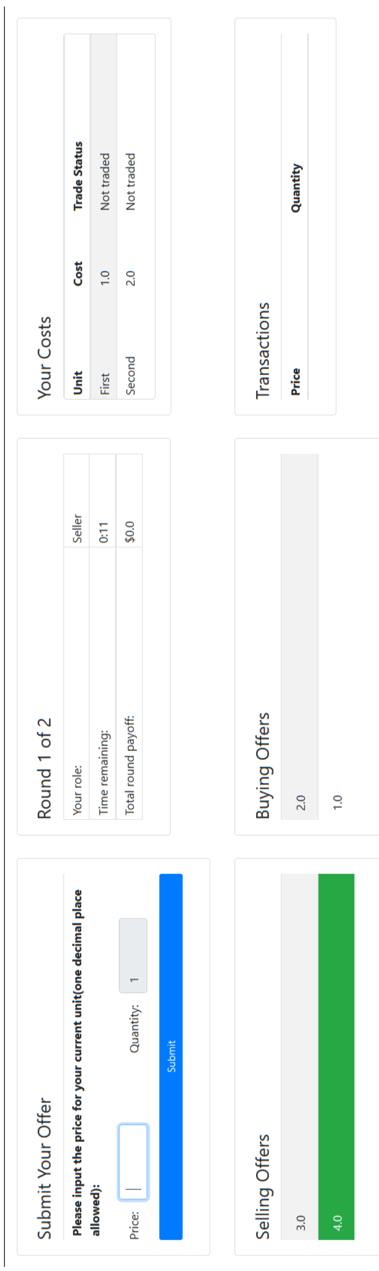
A. 6 B. 2.1 C. 4 D. 5

(b) Which of the following prices can you submit to accept the buying offer of \$3? Circle you answer.

A. 3.5 B. 4.1 C. 5 D. 3

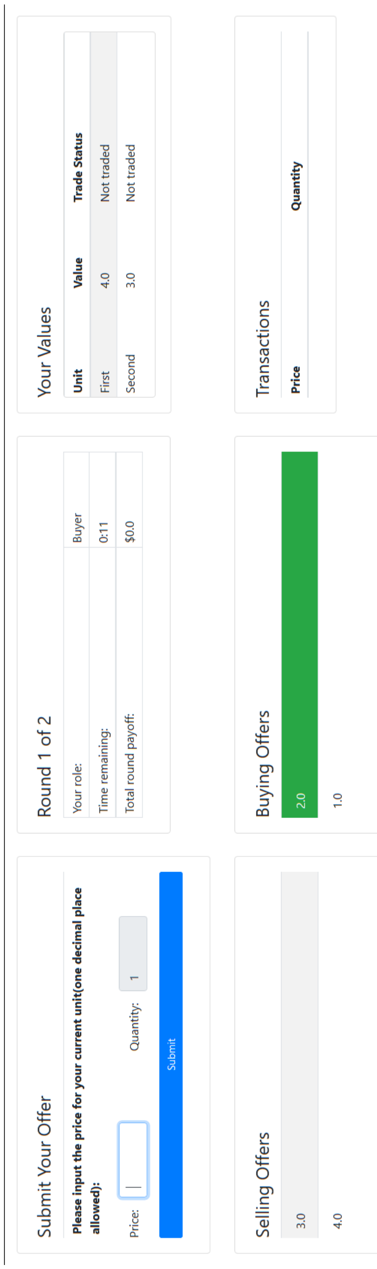
(c) Suppose the first and second unit you sell will cost \$0.1 and \$0.4 respectively, and you accept both buying offers on the list. What is your payoff?

$$\text{Your payoff} = \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Revenue}} - \underbrace{\$\_\_\_\_\_\_\_\_}_{\text{Cost(s)}} = \$\_\_\_\_\_\_\_\_$$



Screen shot 1: Interface for a seller

Note that numbers here are for demonstration purpose.



Screen shot 2: Interface for a buyer

Note that numbers here are for demonstration purpose.

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